Exercise 6.3

Q.1 Use factorization to find the square root of the following expression. **(A.B+U.B)**

(LHR 2014, 15, GRW 2013, FSD 2017, SGD 2016)

(i) $4x^2 - 12xy + 9y^2$

Solution:

$$4x^2 - 12xy + 9y^2 = 4x^2 - 6xy - 6xy + 9y^2$$
$$= 2x(2x - 3y) - 3y(3x - 3y)$$
$$= (2x - 3y)(2x - 3y)$$
$$4x^2 - 12y + 9y^2 = (2x - 3y)^2$$

Taking square root on both sides

$$\sqrt{4x^2 - 12xy + 9y^2} = \sqrt{[2x - 3y]^2}$$
$$= \pm(2x - 3y)$$

(ii) $x^2 - 1 + \frac{1}{4x^2}$

Solution:

$$x^2 - 1 + \frac{1}{4x^2}$$
$$= (x)^2 - 2(x)\left[\frac{1}{2x}\right] + \left[\frac{1}{2x}\right]^2 \qquad \because a^2 - 2ab + b^2 = (a - b)^2$$
$$= \left[x - \frac{1}{2x}\right]^2$$

Taking square root

$$\sqrt{x^2 - 1 + \frac{1}{4x^2}} = \sqrt{\left[x - \frac{1}{2x}\right]^2}$$
$$\sqrt{x^2 - 1 + \frac{1}{4x^2}} = \pm\left(x - \frac{1}{2x}\right)$$

(iii) $\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2$ **(A.B)**

Solution:

$$\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2$$

$$=\left(\frac{1}{4}x\right)^2-2\left(\frac{1}{4}x\right)\left(\frac{1}{6}y\right)+\left(\frac{1}{6}y\right)^2$$

$$=\left(\frac{x}{4}-\frac{y}{6}\right)^2 \qquad \because a^2-2ab+b^2=(a-b)^2$$ **(K.B)**

Taking the square root

$$\sqrt{\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2}=\sqrt{\left(\frac{1}{4}x-\frac{1}{6}y\right)^2}$$

$$=\pm\left(\frac{1}{4}x-\frac{1}{6}y\right)$$

$$=\pm\left(\frac{x}{4}-\frac{y}{6}\right)$$

(iv) $4(a+b)^2-12(a^2+b^2)+9(a-b)^2$

Solution:

$$4(a+b)^2-12(a2+b^2)+9(a-b)^2$$

$$=\left[2(a+b)^2\right]-2\left[2(a+b)\right]\left[3(a-b)\right]+\left[3(a-b)\right]^2$$

$$=\left[2(a+b)-3(a-b)\right]^2 \qquad \because a^2-2ab+b^2=(a-b)^2$$ **(K.B)**

Taking square root

$$\sqrt{4(a+b)^2-12(a^2+b^2)+9(a-b)^2}=\sqrt{\left[2(a+b)-3(a-b)\right]^2}$$

$$=\pm\left[2a+2b-3a+3b\right]$$

$$=\pm(5b-a)$$

(v) $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

Solution:

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$$

$$=\frac{(2x^3)^2-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2+2(3x^2)(4y^2)+(4y^2)^2}$$

$$=\frac{\left[2x^3-3y^3\right]^2}{\left[3x^2+4y^2\right]^2} \qquad \because a^2\mp 2ab+b^2=(a\mp b)^2$$ **(K.B)**

Taking square root

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}}=\frac{\sqrt{\left(2x^3-3y^3\right)^2}}{\sqrt{\left(3x^2+4y^2\right)^2}}$$

$$=\pm\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)$$

(vi) $\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right),(x\neq 0)$

Solution:

$$\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)$$

By adding and subtracting 4

$$=x^2+\frac{1}{x^2}+2-4\left(x-\frac{1}{x}\right)$$

$$=\left(x^2+\frac{1}{x^2}-2\right)+2-4\left(x-\frac{1}{x}\right)+2$$

$$=x^2+\frac{1}{x^2}-2-4\left(x-\frac{1}{x}\right)+4$$

$$=\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right)(2)+(2)^2$$

$$=\left[\left(x-\frac{1}{x}\right)-2\right]^2 \qquad \because a^2-2ab+b^2=(a-b)^2 \qquad \textbf{(A.B+K.B)}$$

Taking square root

$$\sqrt{\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)}=\sqrt{\left[x-\frac{1}{x}-2\right]^2}$$

$$=\pm\left(x-\frac{1}{x}-2\right)$$

(vii) $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$

Solution:

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$$

$$=\left[x^2+\frac{1}{x^2}\right]^2-4\left[x^2+\frac{1}{x^2}+2\right]+12$$

$$=\left[x^2+\frac{1}{x^2}\right]^2-4x^2-\frac{4}{x^2}-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+4$$

$$=\left[x^2+\frac{1}{x^2}\right]^2-2\left[x^2+\frac{1}{x^2}\right](2)+(2)^2 \qquad \because a^2-2ab+b^2=(a-b)^2$$

$$=\left[x^2+\frac{1}{x^2}-2\right]^2$$

Taking square root

$$\sqrt{\left[x^2+\frac{1}{x^2}\right]^2-4\left[x+\frac{1}{x}\right]^2+12}=\sqrt{\left[x^2+\frac{1}{x^2}-2\right]^2}$$

$$=\pm\left(x^2+\frac{1}{x^2}-2\right)$$

(viii) $\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)$

Solution:

$$\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)$$

$$=\left[x^2+2x+x+2\right]\left[x^2+3x+x+3\right]\left[x^2+3x+2x+6\right]$$

$$=\left[x(x+2)+1(x+2)\right]\left[x(x+3)+1(x+3)\right]\left[x(x+3)+2(x+3)\right]$$

$$=(x+2)(x+1)(x+3)(x+1)(x+3)(x+2)$$

$$=(x+2)^2(x+1)^2(x+3)^2$$

Taking square root

$$\sqrt{(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)}=\sqrt{(x+2)^2(x+1)^2(x+3)^2}$$

$$=\pm(x+1)(x+2)(x+3)$$

(ix) $\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)$

Solution:

$$\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)$$

$$=(x^2+7x+1x+7)(2x^2-3x+2x-3)(2x^2+14x-3x-21)$$

$$=\left[(x(x+7)+1(x+7)\right]\left[x(2x-3)+1(2x-3)\right]\left[(2x(x+7)-3(x+7)\right]$$

$$=(x+7)(x+1)(2x-3)(x+1)(x+7)(2x-3)$$

$$=(x+7)^2(x+1)^2(2x-3)^2$$

Taking square root

$$\sqrt{(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)} = \sqrt{(x+7)^2(x+1)^2(2x-3)^2}$$

$$= \pm(x+1)(x+7)(2x-3)$$

Q.2 Use division method to find the square root of the following expression. **(K.B)**

(i) $4x^2+12xy+9y^2+16x+24y+16$

Solution:

$4x^2+12xy+9y^2+16x+24y+16$

$$2x+3y+4$$

$$2x\,\big)\,\cancel{4x^2}+12xy+9y^2+16x+24y+16$$

$$\pm\cancel{4x^2}$$

$$4x+3y\,\big)\,12\cancel{xy}+\cancel{9y^2}+16x+24y+16$$

$$\pm 12\cancel{xy}\pm\cancel{9y^2}$$

$$4x+6y+4\,\big)\,\cancel{16x}+\cancel{24y}+\cancel{16}$$

$$\cancel{16x}\pm\cancel{24y}\pm\cancel{16}$$

$$0$$

Square root = $\pm(2x+3y+4)$

(ii) $x^4-10x^3+37x^2-60x+36$ **(A.B+K.B)**

Solution: $x^4-10x^3+37x^2-60x+36$

$$x^2-5x+6$$

$$x^2\,\big)\,\cancel{x^4}-10x^3+37x^2-60x+36$$

$$\pm\cancel{x^4}$$

$$2x^2-5x\,\big)\,-\cancel{10x^3}+37x^2-60x+36$$

$$\pm\cancel{10x^3}\pm 25x^2$$

$$2x^2-10x+6\,\big)\,\cancel{12x^2}-\cancel{60x}+\cancel{36}$$

$$\pm\cancel{12x^2}\mp\cancel{60x}\pm\cancel{36}$$

$$\times$$

Square root $=\pm(x^2-5x+6)$

(iii) $9x^4-6x^3+7x^2-2x+1$

Solution:

$$9x^4 - 6x^3 + 7x^2 - 2x + 1$$

$$\begin{array}{r|l}
 & 3x^2 - x + 1 \\
3x^2 & \cancel{9x^4} - 6x^3 + 7x^2 - 2x + 1 \\
 & \pm \cancel{9x^4} \\
6x^2 - x & \cancel{-6x^3} + 7x^2 - 2x + 1 \\
 & \pm \cancel{6x^3} \pm x^2 \\
6x^2 - 2x + 1 & \cancel{6x^2} - \cancel{2x} + \cancel{1} \\
 & \pm \cancel{6x^2} \mp \cancel{2x} \pm \cancel{1} \\
 & \times
\end{array}$$

Square root $\pm\left(3x^2 - x + 1\right)$

(iv) $4 + 25x^2 + 7x^2 - 2x + 1$ **(A.B)**

Solution: $4 + 25x^2 - 12x - 24x^3 + 16x^4$

$$\begin{array}{r|l}
 & 4x^2 - 3x + 2 \\
4x^2 & \cancel{16x^4} - 24x^3 + 25x^2 - 12x + 4 \\
 & \pm \cancel{16x^4} \\
8x^2 - 3x & -\cancel{24x^3} + 25x^2 - 12x + 4 \\
 & \mp \cancel{24x^3} \pm 9x^2 \\
8x^2 - 6x + 2 & \cancel{16x^2} - \cancel{12x} + \cancel{4} \\
 & \pm \cancel{16x^2} \mp \cancel{12x} \pm \cancel{4} \\
 & \times
\end{array}$$

Square root $= \pm\left(4x^2 - 3x + 2\right)$

(v) $\dfrac{x^2}{y^2} - \dfrac{10x}{y} + 27 - \dfrac{10y}{x} + \dfrac{y^2}{x^2}, (x \neq 0, y \neq 0)$

Solution: $\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2},(x\neq 0, y\neq 0)$

$$\frac{x}{y}-5+\frac{y}{x}$$

$$\frac{x}{y}\Big)\ \frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2},$$

$$\pm\frac{x^2}{y^2}$$

$$\frac{2x}{y}-5\Big)\ -\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2}$$

$$\mp\frac{10x}{y}\pm 25$$

$$\frac{2x}{y}-10+\frac{y}{x}\Big)\ +2-\frac{10y}{c}+\frac{y^2}{x^2}$$

$$\pm 2\mp\frac{10y}{x}\pm\frac{y^2}{x^2}$$

$$\times$$

Square root $=\pm\left(\frac{x}{y}-5+\frac{y}{x}\right)$

Q.3 Find the value of k for which the following expressions will become a perfect square.

(i) $4x^4-12x^3+37x^2-42x+k$ **(A.B)**

Solution: $4x^4-12x^3+37x^2-42x+k$

$$2x^2-3x+7$$

$$2x^2\Big)\ 4x^4-12x^3+37x^2-42x+k$$

$$\pm 4x^4$$

$$4x^2-3x\Big)\ -12x^3+37x^2-42x+k$$

$$-12x^3\pm 9x^2$$

$$4x^2-6x+7\Big)\ 28x^2-42x+k$$

$$\pm 28x^2\mp 42x\pm 49$$

$$k-49$$

In the case of perfect square remainder is always is equal to zero so

$k-49=0$

$k=49$

(ii) $x^4-4x^3+10x^2-kx+9$

Solution:

$x^4-4x^3+10x^2-kx+9$

$$x^2\overline{)x^4-4x^3+10x^2-kx+9}\quad x^2-2x+3$$
$$\pm x^4$$
$$2x^2-2x\overline{)-4x^3+10x^2-kx+9}$$
$$\mp 4x^3 \pm 4x^2$$
$$2x^2-4x+3\overline{)6x^2-kx+9}$$
$$-6x^2 \mp 12x \pm 9$$
$$-kx+12x=0$$

In the case of square root remainder is always equal to zero

$k-12=0$

$k=12$

Q.4 Find the value of l and m for which the following expression will be perfect square

(i) $x^4+4x^3+16x^2+lx+m$ **(A.B)**

Solution:

$x^4+4x^3+16x^2+lx+m$

$$x^2\overline{)x^4+4x^3+16x^2+lx+m}\quad x^2+2x+6$$
$$\mp x^4$$
$$2x^2+2x\overline{)4x^3+16x^2+lx+m}$$
$$\pm 4x^3 \pm 4x^2$$
$$2x^2+4x+6\overline{)12x^2+lx+m}$$
$$\pm 12x^2 \pm 24x \pm 36$$
$$(l-24)x=0,\quad m-36=0$$

In the case of square root remainder is always zero

$l-24=0$, $m-36=0$

$l=24$, $m=36$

(ii) $49x^4-70x^3+109x^2+lx-m$

Solution:

$49x^4-70x^3+109x^2+lx-m$

$$\begin{array}{l} \quad 7x^2-5x+6 \\ 7x^2\,\big)\,49x^4-70x^3+109x^2+lx-m \\ \quad \pm 49x^4 \\ \hline 14x^2-5x\,\big)\,-70x^3+109x^2+lx-m \\ \quad \mp 70x^3 \pm 25x^2 \\ \hline 14x^2-10x+6\,\big)\,84x^2+lx-m \\ \quad \pm 84x^2 \mp 60x \pm 36 \\ \hline \quad lx+60x-m-36 \\ \quad (l+60)x-m-36 \end{array}$$

In the case of square root remainder is always equal to zero

$l+60=0$ $-m-36=0$

$l+60=0$, $-m=36$

$l=-60$, $m=-36$

Q.5 **To make the expression** $9x^4-12x^3+22x^2-13x+12$ **a perfect square** **(A.B)**

Solution: $9x^4-12x^3+22x^2-13x+12$

$$\begin{array}{l} \quad 3x^2-2x+3 \\ =3x^2\,\big)\,9x^4-12x^3+22x^2-13x+12 \\ \quad \pm 9x^4 \\ \hline 6x^2-2x\,\big)\,-12x^3+22x^2-13x+12 \\ \quad \mp 12x^3 \pm 4x^2 \\ \hline 6x^2-4x+3\,\big)\,18x^2-13x+12 \\ \quad \pm 18x^2 \mp 12x \pm 9 \\ \hline \quad -x+3 \end{array}$$

(i) $+x-3$ **is to be added**

(ii) $-x+3$ **is to be subtract from it**

(iii) $-x+3=0$

$x=3$