Mathematics-9
Unit 9 – Exercise 9.2

Exercise 9.2

Q.1 Show whether the points with vertices (5,-2),(5,4) and (-4,1) are the vertices of an equilateral triangle or an isosceles triangle $P(5,-2), Q(5,4), R(-4,1)$ **(A.B)**

Solution:

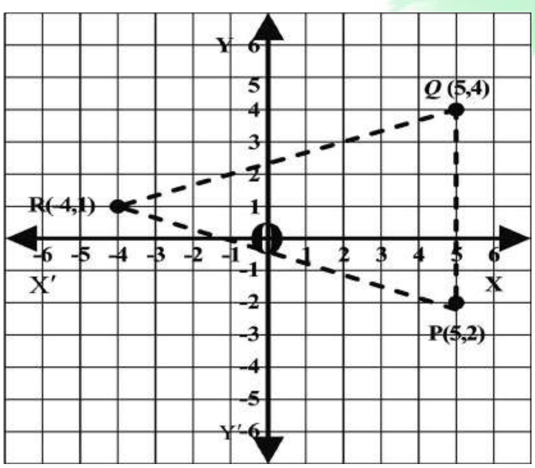
We know that the distance formula is

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

We have $P(5,-2), Q(5,4)$

$$|PQ| = \sqrt{|5-5|^2 + |4-(-2)|^2}$$

$$|PQ| = \sqrt{(0)^2 + (4+2)^2}$$



$$|PQ| = \sqrt{(6)^2}$$

$$|PQ| = 6$$

$$|QR| = \sqrt{|-4-5|^2 + |1-4|^2}$$

$$|QR| = \sqrt{(-9)^2 + (-3)^2}$$

$$|QR| = \sqrt{81+9}$$

$$|QR| = \sqrt{90}$$

$$|QR| = \sqrt{9\times 10} = 3\sqrt{10}$$

$$|RP| = \sqrt{|5-(-4)|^2 + |-2-1|^2}$$

$$|RP| = \sqrt{(5+4)^2 + (-3)^2}$$

$$|RP| = \sqrt{(9)^2 + 9}$$

$$|RP| = \sqrt{81+9}$$

$$|RP| = \sqrt{90}$$

$$|RP| = \sqrt{9\times 10} = 3\sqrt{10}$$

As $|QR| = |PR|$

Lengths of two sides of triangle are equal, so it is an isosceles triangle.

Q.2 Show whether or not the points with vertices $(-1,1),(5,4),(2,-2)$ **and** $(-4,1)$ **form a Square? (A.B)**

Solution:

$$P(-1,1)Q(5,4)R(2,-2)S(-4,1)$$

Distance $= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$$|PQ| = \sqrt{\left|5-(-1)^2\right| + |4-1|^2}$$

$$|PQ| = \sqrt{|5+1|^2 + |3|^2}$$

$|PQ| = \sqrt{6^2 + 9}$

$|PQ| = \sqrt{36 + 9}$

$|PQ| = \sqrt{45}$

$|PQ| = \sqrt{9 \times 5}$

$|PQ| = 3\sqrt{5}$

$|QR| = \sqrt{|2-5|^2 + |-2-4|^2}$

$|QR| = \sqrt{(-3)^2 + (-6)^2}$

$|QR| = \sqrt{9 + 36}$

$|QR| = \sqrt{45}$

$|QR| = \sqrt{9 \times 5}$

$|QR| = 3\sqrt{5}$

$|RS| = \sqrt{|-4-2|^2 + |1-(-2)|^2}$

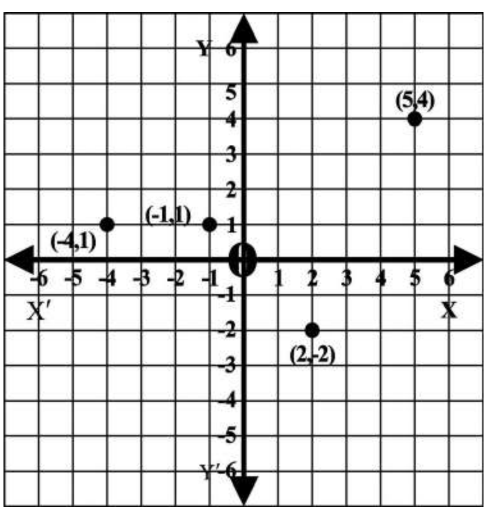
$|RS| = \sqrt{(-6)^2 + (1+2)^2} = \sqrt{36 + (3)^2}$

$|RS| = \sqrt{36 + 9}$

$|RS| = \sqrt{45}$

$|RS| = \sqrt{9 \times 5}$

$|RS| = 3\sqrt{5}$



$|SP| = \sqrt{|-4-(-1)|^2 + |1-1|^2}$

$|SP| = \sqrt{(-4+1)^2 + (0)^2}$

$|SP| = \sqrt{(-3)^2}$

$|SP| = \sqrt{9}$

$|SP| = 3$

If all the lengths are same then it will be a Square, all the lengths are not equal so it is not square.

$|PQ| = |QR| = |RS| \neq |SP|$

Q.3 Show whether or not the points with coordinates $(1,3), (4,2)$ and $(-2,6)$ are vertices of a right triangle? (A.B)

Solution:

$A(1,3), B(4,2), C(-2,6)$

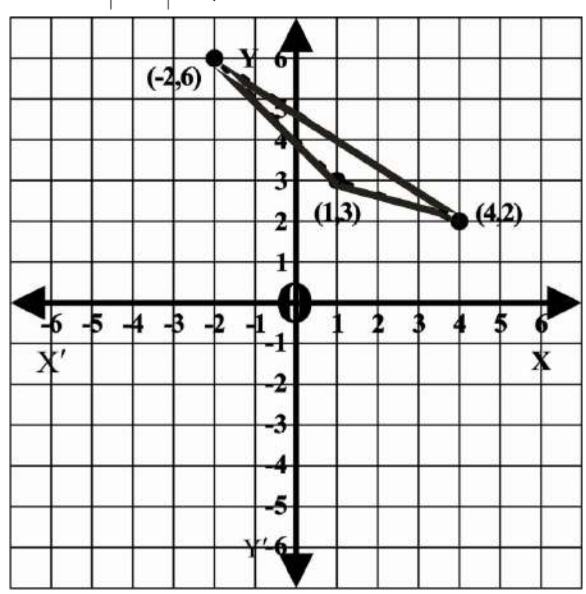
$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|4-1|^2 + |2-3|^2}$

$|AB| = \sqrt{(3)^2 + (-1)^2}$

$|AB| = \sqrt{9 + 1}$

$|AB| = \sqrt{10}$



$|BC| = \sqrt{|-2-4|^2 + |6-2|^2}$

$|BC| = \sqrt{(-6)^2 + (4)^2}$

$|BC| = \sqrt{36+16}$

$|BC| = \sqrt{52}$

$|CA| = \sqrt{|-2-1|^2 + |6-3|^2} = \sqrt{(-3)^2 + (3)^2}$

$|CA| = \sqrt{9+9}$

$|CA| = \sqrt{18}$

By Pythagoras theorem

$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$

$(\sqrt{52})^2 = (\sqrt{18})^2 + (\sqrt{10})^2$

$52 = 18 + 10$

52=28

Since $52 \neq 28$

So it is not right angle triangle.

Q.4 Use distance formula to prove whether or not the points $(1,1), (-2,-8)$ and (4,10) lie on a straight line? (A.B)

Solution:

$A(1,1), B(-2,-8), C(4,10)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|-2-1|^2 + |-8-1|^2}$

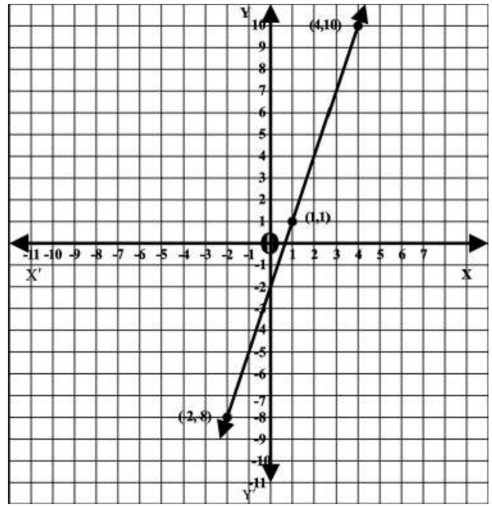
$|AB| = \sqrt{(-3)^2 + (-9)^2}$

$|AB| = \sqrt{9+81}$

$|AB| = \sqrt{90}$

$|AB| = \sqrt{9 \times 10}$

$|AB| = 3\sqrt{10}$



$|BC| = \sqrt{|4-(-2)|^2 + |10-(-8)|^2}$

$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$

$|BC| = \sqrt{(6)^2 + (18)^2}$

$|BC| = \sqrt{36+324}$

$|BC| = \sqrt{360}$

$|BC| = \sqrt{36 \times 10}$

$|BC| = 6\sqrt{10}$

$|AC| = \sqrt{|4-1|^2 + |10-1|^2}$

$|AC| = \sqrt{(3)^2 + (9)^2}$

$|AC| = \sqrt{9+81}$

$|AC| = \sqrt{90}$

$|AC| = \sqrt{9 \times 10}$

$|AC| = 3\sqrt{10}$

$|AC| + |AB| = |BC|$

$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$

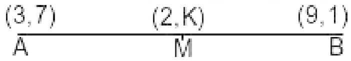
$6\sqrt{10} = 6\sqrt{10}$

It means that they lie on same line so they are collinear.

Q.5 Find K given that point (2, K) is equidistance from (3, 7) and (9,1)

Solution:

$M(2,K), A(3,7)$ and $B(9,1)$



$$|\overline{AM}| = |\overline{BM}|$$

$$\sqrt{|2-3|^2 + |K-7|^2} = \sqrt{|9-2|^2 + |1-K|^2}$$

$$\sqrt{(-1)^2 + (K-7)^2} = \sqrt{(7)^2 + (1-K)^2}$$

Taking square on both Sides

$$\left(\sqrt{1+K^2+49-14K}\right)^2 = \left(\sqrt{49+1+K^2-2K}\right)^2$$

$$K^2 - 14K + 50 = 50 + K^2 - 2K$$

$$\cancel{K^2} - 14K \cancel{+50} \cancel{-50} \cancel{-K^2} + 2K = 0$$

$$-12K = 0$$

$$K = \frac{0}{-12}$$

$$K = 0$$

Q.6 Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ are Collinear. (A.B)

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|3-0|^2 + |-5-7|^2}$$

$$|AB| = \sqrt{(3)^2 + (-12)^2}$$

$$|AB| = \sqrt{9+144}$$

$$|AB| = \sqrt{153}$$

$$|AB| = \sqrt{9 \times 17}$$

$$|AB| = 3\sqrt{17}$$

$$|BC| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$$

$$|BC| = \sqrt{(-5)^2 + (15+5)^2}$$

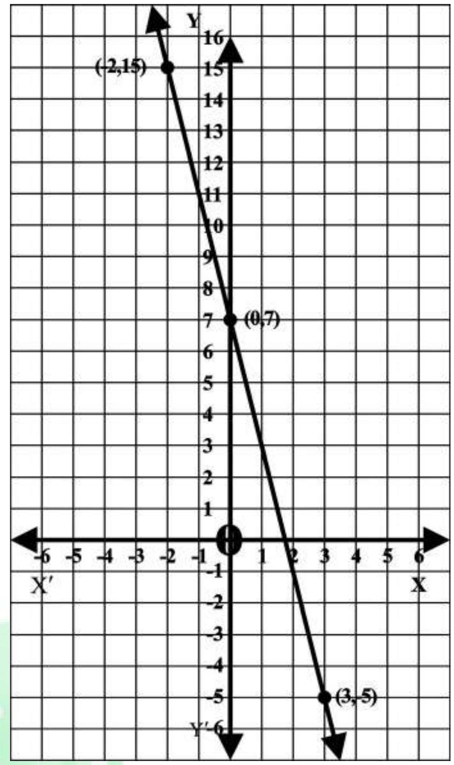
$$|BC| = \sqrt{25 + (20)^2}$$

$$|BC| = \sqrt{25+400}$$

$$|BC| = \sqrt{425}$$

$$|BC| = \sqrt{25 \times 17}$$

$$|BC| = 5\sqrt{17}$$



$$|AC| = \sqrt{|-2-0|^2 + |15-7|^2}$$

$$|AC| = \sqrt{(-2)^2 + (8)^2}$$

$$|AC| = \sqrt{4+64}$$

$$|AC| = \sqrt{68}$$

$$|AC| = \sqrt{4 \times 17}$$

$$|AC| = 2\sqrt{17}$$

$$|AB| + |AC| = |BC|$$

$$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$$

$$5\sqrt{17} = 5\sqrt{17}$$

L.H.S = R.H.S

So

They lie on same line and they are collinear.

Q.7 **Verify whether or not the points $O(0,0), A(\sqrt{3},1), B(\sqrt{3},-1)$ are the vertices of an equilateral triangle.** **(A.B)**

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|OA| = \sqrt{|\sqrt{3} - 0|^2 + |1 - 0|^2}$$

$$|OA| = \sqrt{(\sqrt{3})^2 + (1)^2}$$

$$|OA| = \sqrt{3+1}$$

$$|OA| = \sqrt{4}$$

$$|OA| = 2$$

$$|OB| = \sqrt{|\sqrt{3} - 0|^2 + |-1 - 0|^2}$$

$$|OB| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|OB| = \sqrt{3+1}$$

$$|OB| = \sqrt{4}$$

$$|OB| = 2$$

$$|AB| = \sqrt{|\cancel{\sqrt{3}} - \cancel{\sqrt{3}}|^2 + |-1 - 1|^2}$$

$$|AB| = \sqrt{0 + (-2)^2}$$

$$|AB| = \sqrt{4}$$

$$|AB| = 2$$

All the sides are same in length so it is equilateral triangle.

Q.8 **Show that the points $A(-6,-5)$, $B(5,-5)$, $C(5,-8)$ and $D(-6,-8)$ are the vertices of a rectangle. Find the lengths of its diagonals. Are they equal?** **(K.B + A.B)**

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|5 - (-6)|^2 + |-5 - (-5)|^2}$$

$$|AB| = \sqrt{(5+6)^2 + (-5+5)^2}$$

$$|AB| = \sqrt{(11)^2 + (0)^2} = \sqrt{121}$$

$$|AB| = 11$$

$$|BC| = \sqrt{|5 - 5|^2 + |-8 - (-5)^2|}$$

$$|BC| = \sqrt{(0)^2 + (-8+5)^2} = \sqrt{(-3)^2}$$

$$|BC| = \sqrt{(-3)^2} = \sqrt{9}$$

$$|BC| = 3$$

$$|DC| = \sqrt{|-6 - 5|^2 + |-8 - (-8)|^2}$$

$$|DC| = \sqrt{(-11)^2 + (-8+8)^2}$$

$$|DC| = \sqrt{121 + 0} = \sqrt{121}$$

$$|DC| = 11$$

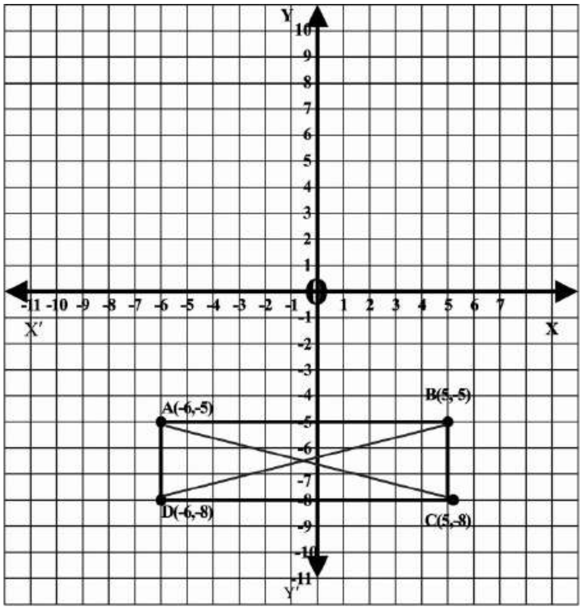
$$|DA| = \sqrt{|-6 - (-6)|^2 + |-5 - (-8)|^2}$$

$$|DA| = \sqrt{(-6+6)^2 + (-5+8)^2}$$

$$|DA| = \sqrt{(0)^2 + (3)^2} = \sqrt{0+9}$$

$$|DA| = \sqrt{9}$$

$$|DA| = 3$$



Diagonal distance of

$|A\ C|$ and $|B\ D|$

$$|A\ C| = \sqrt{|5-(-6)|^2 + |-8-(-5)|^2}$$

$$|A\ C| = \sqrt{(5+6)^2 + (-8+5)^2}$$

$$|A\ C| = \sqrt{(11)^2 + (-3)^2}$$

$$|A\ C| = \sqrt{121+9}$$

$$|A\ C| = \sqrt{130}$$

$$|B\ D| = \sqrt{|-6-5|^2 + |-8-(-5)|^2}$$

$$|B\ D| = \sqrt{(-11)^2 + (-8+5)^2}$$

$$|B\ D| = \sqrt{121+(-3)^2} = \sqrt{121+9}$$

$$|B\ D| = \sqrt{130}$$

By Pythagoras theorem

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$(\text{AC})^2 = (DC)^2 + (\text{AD})^2$$

$$\left(\sqrt{130}\right)^2 = (11)^2 + (3)^2$$

130 = 121 + 9

130 = 130

L.H.S = R.H.S

Lengths of both diagonals are same it is a rectangle.

Q.9 Show that the point $M(-1, 4), N(-5, 3), P(1,-3)$ and $Q(5,-2)$ are vertices of a parallelogram. (K.B + A.B)

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|M\ N| = \sqrt{|-5-(-1)|^2 + |3-4|^2}$$

$$|M\ N| = \sqrt{(-5+1)^2 + (-1)^2}$$

$$|M\ N| = \sqrt{(-4)^2 + 1} = \sqrt{16+1}$$

$$|M\ N| = \sqrt{17}$$

$$|N\ P| = \sqrt{|1-(-5)|^2 + |-3-3|^2}$$

$$|N\ P| = \sqrt{(1+5)^2 + (-6)^2}$$

$$|N\ P| = \sqrt{(6)^2 + (6)^2} = \sqrt{36+36}$$

$$|N\ P| = \sqrt{72}$$

$$|N\ P| = \sqrt{36\times 2}$$

$$|N\ P| = 6\sqrt{2}$$

$$|P\ Q| = \sqrt{|5-1|^2 + |-2-(-3)|^2}$$

$$|P\ Q| = \sqrt{(4)^2 + (-2+3)^2}$$

$$|P\ Q| = \sqrt{16+(1)^2}$$

$$|P\ Q| = \sqrt{16+1}$$

$$|P\ Q| = \sqrt{17}$$

$$|M\ Q| = \sqrt{|5-(-1)|^2 + |-2-4|^2}$$

$$|M\ Q| = \sqrt{(5+1)^2 + (-6)^2}$$

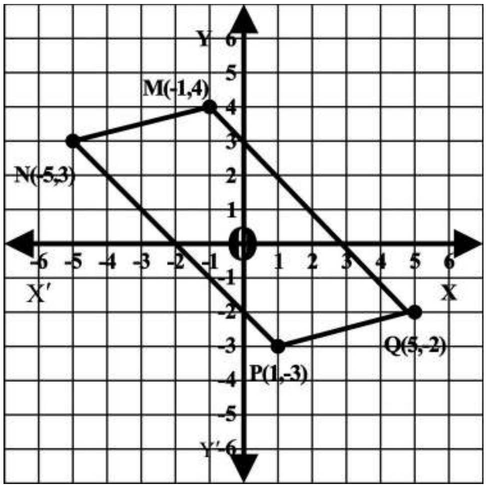
$$|M\ Q| = \sqrt{(6)^2 + 36}$$

$$|M\ Q| = \sqrt{36+36}$$

$$|M\ Q| = \sqrt{72}$$

$$|M\ Q| = \sqrt{36\times 2}$$

$$|M\ Q| = 6\sqrt{2}$$



Length of a diagonal

$|NQ| = \sqrt{|5-(-5)|^2 + |-2-3|^2}$

$|NQ| = \sqrt{(5+5)^2 + (-5)^2}$

$|NQ| = \sqrt{(10)^2 + (25)}$

$|NQ| = \sqrt{100+25} = \sqrt{125}$

$m\angle N = 90^o$, if

$(QM)^2 + (MN)^2 = (QN)^2$

$\left(6\sqrt{2}\right)^2 + \left(\sqrt{17}\right)^2 = \left(\sqrt{125}\right)^2$

$36 \times 2 + 17 = 125$

72 + 17 = 125

89 = 125

They are not equal, so it is not right angle.

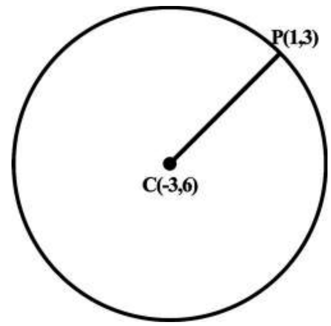
But $|MN| = |PQ|$ and $|NP| = |MQ|$

Opposite side are equal so it is a Parallelogram.

Q.10 Find the length of the diameter of the circle having centre at $C(-3,6)$ and passing through P (1, 3).

(K.B + A.B)

Solution:



CP is the radius of a circle

So

$|CP| = \sqrt{|-3-1|^2 + |6-3|^2}$

$|CP| = \sqrt{(-4)^2 + (3)^2}$

$|CP| = \sqrt{16+9}$

$|CP| = \sqrt{25}$

$|CP| = 5$

Diameter = 2 radius

Diameter = 2(CP)

Diameter = 2(5)

Diameter = 10 units.

And

$|M_1M_3| = \sqrt{(1-1)^2 + (2-5)^2} = \sqrt{0^2 + (-3)^2} = 3$

All the lengths of three sides are different. Hence the triangle $M_1M_2M_3$ is a scalene triangle.