Exercise 9.3

Q.1 Find the midpoint of the line Segments joining each of the following pairs of points

(K.B + A.B)

Solution:

(a) $A(9,2), B(7,2)$

(MTN 2016, 17, SGD 2017, D.G.K 2016)

Let $M(x,y)$ be the midpoint of AB,
Then by midpoint formula

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{9+7}{2},\frac{2+2}{2}\right)$$

$$=M\left(\frac{\cancel{16}^{8}}{\cancel{2}},\frac{\cancel{4}^{2}}{\cancel{2}}\right)$$

$$=M(8,2)$$

(b) $A(2,-6), B(3,-6)$

(LHR 2017, MTN 2014, 15, 17, SWL 2015, SGD 2013, D.G.K 2014)

Let $M(x,y)$ be the midpoint of AB then by Midpoint formula

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{2+3}{2},\frac{-6-6}{2}\right)$$

$$M(x,y)=M\left(\frac{5}{2},\frac{-\cancel{12}^{6}}{\cancel{2}}\right)$$

$$M(x,y)=M(2.5,-6)$$

(c) $A(-8,1), B(6,1)$

(LHR 2017, GRW 2017)

Let $M(x,y)$ be the midpoint of AB then by Midpoint formula

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{-8+6}{2},\frac{1+1}{2}\right)$$

$$M(x,y)=M\left(\frac{-\cancel{2}}{\cancel{2}},\frac{\cancel{2}}{\cancel{2}}\right)$$

$$M(x,y)=M(-1,1)$$

(d) A (-4 , 9) , B (-4 , -3)

Let $M(x,y)$ be the midpoint of AB then by Midpoint formula

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{-4-4}{2},\frac{9-3}{2}\right)$$

$$M(x,y)=M\left(\frac{-\cancel{8}^{4}}{\cancel{2}},\frac{\cancel{6}^{3}}{\cancel{2}}\right)$$

$$M(x,y)=M(-4,3)$$

(e) $A(3,-11), B(3,-4)$

(LHR 2017, GRW 2014, MTN 2014, 15, SGD 2015, SWL 2015, 16, FSD 2017, BWP 2016)

Let $M(x,y)$ is the midpoint of AB

$$M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{3+3}{2},\frac{-11-4}{2}\right)$$

$$M(x,y)=M\left(\frac{6}{2},\frac{-15}{2}\right)$$

$M(x,y)=M(3,-7.5)$

(f) A (0, 0) , B (0, -5)

(LHR 2017, GRW 2016, FSD 2016, SWL 2014, D.G.K 2015)

Let $M(x,y)$ is the midpoint of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

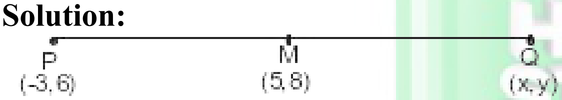
$$M(x,y)=M\left(\frac{0+0}{2},\frac{0-5}{2}\right)$$

$$M(x,y)=M\left(\frac{0}{2},\frac{-5}{2}\right)$$

$$=M(0,-2.5)$$

Q.2 The end point P of a line segment PQ is $(-3,6)$ and its midpoint is $(5,8)$ find the coordinates of the end point Q.
(FSD 2017) (K.B + U.B)

Solution:



Let Q be the point (x,y), $M(5,8)$ is the midpoint of PQ

$$M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$x=\frac{x_1+x_2}{2}$$

$$5=\frac{-3+x}{2}$$

$$5\times 2=-3+x$$

$$10+3=x$$

$$x=13$$

$$y=\frac{y_1+y_2}{2}$$

$$8=\frac{6+y}{2}$$

$$2\times 8=6+y$$

$$16-6=y$$

$$y=10$$

Hence point Q is $(13,10)$

Q.3 Prove that midpoint of the hypotenuse of a right triangle is equidistant from it three vertices $P(-2,5), Q(1,3)$ and $R(-1,0)$.
(K.B + A.B + U.B)

Solution:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

$$|PQ|=\sqrt{|-2-1|^2+|5-3|^2}$$

$$|PQ|=\sqrt{(-3)^2+(2)^2}$$

$$|PQ|=\sqrt{9+4}$$

$$|PQ|=\sqrt{13}$$

$$|QR|=\sqrt{|1-(-1)|^2+|3-0|^2}$$

$$|QR|=\sqrt{(1+1)^2+(3)^2}$$

$$|QR|=\sqrt{(2)^2+9}=\sqrt{4+9}$$

$$|QR|=\sqrt{13}$$

$$|PR|=\sqrt{|-2-(-1)|^2+|5-0|^2}$$

$$|PR|=\sqrt{|-2+1|^2+|5|^2}$$

$$|PR|=\sqrt{(-1)^2+(5)^2}=\sqrt{1+25}$$

$$|PR|=\sqrt{26}$$

whether it is right angle triangle or not, we use the Pythagoras theorem

$$(PR)^2=(PQ)^2+(QR)^2$$

$$\left(\sqrt{26}\right)^2=\left(\sqrt{13}\right)^2+\left(\sqrt{13}\right)^2$$

26 = 13 + 13

26 = 26 (Satisfied)

It is a right angle triangle and PR is hypotenuse.

Midpoint of PR

$$M(x,y)=\left(\frac{-2-1}{2},\frac{5+0}{2}\right)$$

$$M(x,y)=\left(\frac{-3}{2},\frac{5}{2}\right)$$

$$|MP|=|MR|$$

(i)

$$|MP|=\sqrt{\left|\frac{-3}{2}-(-2)\right|^2+\left|\frac{5}{2}-5\right|^2}$$

$$=\sqrt{\left(\frac{-3}{2}+2\right)^2+\left(\frac{5-10}{2}\right)^2}$$

$$|MP|=\sqrt{\left(\frac{-3+4}{2}\right)^2+\left(\frac{-5}{2}\right)^2}$$

$$=\sqrt{\left(\frac{1}{2}\right)^2+\frac{25}{4}}$$

$$|MP|=\sqrt{\frac{1}{4}+\frac{25}{4}}=\sqrt{\frac{1+25}{4}}$$

$$|MP|=\sqrt{\frac{26}{4}}$$

$$|MP|=\frac{\sqrt{26}}{2}$$

(ii)

$$|M\ R|=\sqrt{\left|\frac{-3}{2}-(-1)\right|^2+\left|\frac{5}{2}-0\right|^2}$$

$$|M\ R|=\sqrt{\left(\frac{-3}{2}+1\right)^2+\left(\frac{5}{2}\right)^2}$$

$$|M\ R|=\sqrt{\left(\frac{-3+2}{2}\right)^2+\frac{25}{4}}$$

$$=\sqrt{\left(\frac{-1}{2}\right)^2+\frac{25}{4}}$$

$$|M\ R|=\sqrt{\frac{1}{4}+\frac{25}{4}}$$

$$|M\ R|=\sqrt{\frac{1+25}{4}}=\sqrt{\frac{26}{4}}$$

$$|M\ R|=\frac{\sqrt{26}}{2}$$

(iii) $M\left(\frac{-3}{2},\frac{5}{2}\right)$, $Q(1,3)$

$$|MQ|=\sqrt{\left(\frac{-3}{2}-1\right)^2+\left(\frac{5}{2}-3\right)^2}$$

$$=\sqrt{\left(\frac{-3-2}{2}\right)^2+\left(\frac{5-6}{2}\right)^2}$$

$$=\sqrt{\left(\frac{-5}{2}\right)^2+\left(\frac{-1}{2}\right)^2}$$

$$=\sqrt{\frac{25}{4}+\frac{1}{4}}=\sqrt{\frac{26}{4}}$$

Hence proved

Q.4 **If** $O(0,0)$, $A(3,0)$ **and** $B(3,5)$ **are three points in the plane find M_1 and M_2 as midpoint of the line segments** AB **and** OB **respectively find** $|M_1M_2|$. **(K.B + U.B)**

Solution:

M_1 is the midpoint of AB

$$M_1(x,y)=M_1\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=M_1\left(\frac{3+3}{2},\frac{0+5}{2}\right)$$

$$=M_1\left(\frac{6}{2},\frac{5}{2}\right)$$

$$=M_1\left(3,\frac{5}{2}\right)$$

M_2 is the midpoint of OB

$$M_2\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=M_2\left(\frac{0+3}{2},\frac{0+5}{2}\right)$$

$$= M_2\left(\frac{3}{2}, \frac{5}{2}\right)$$

Now

$$|M_1M_2| = \sqrt{\left|\frac{3}{2} - 3\right|^2 + \left|\frac{5}{2} - \frac{5}{2}\right|^2}$$

$$|M_1M_2| = \sqrt{\left(\frac{3-6}{2}\right)^2 + (0)^2}$$

$$= \sqrt{\left(\frac{-3}{2}\right)^2 + 0}$$

$$|M_1M_2| = \sqrt{\frac{9}{4}}$$

$$|M_1M_2| = \frac{3}{2}$$

Q.5 Show that the diagonals of the parallelogram having vertices $A(1,2), B(4,2), C(-1,-3)$ and $D(-4,-3)$ bisect each other.

(K.B + A.B + U.B)

Solution:

$ABCD$ is parallelogram with vertices $A(1,2), B(4,2), C(-1,-3), D(-4,-3)$.

Let $\overline{BD}$ and $\overline{AC}$ the diagonals of parallelogram intersect at point M.

Finding midpoint of AC

Using midpoint formula

$$M_1(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M_1(x,y) = M_1\left(\frac{1-1}{2}, \frac{2-3}{2}\right)$$

$$M_1(x,y) = M_1\left(\frac{0}{2}, \frac{-1}{2}\right) = \left(0, \frac{-1}{2}\right)$$

Midpoint of BD,

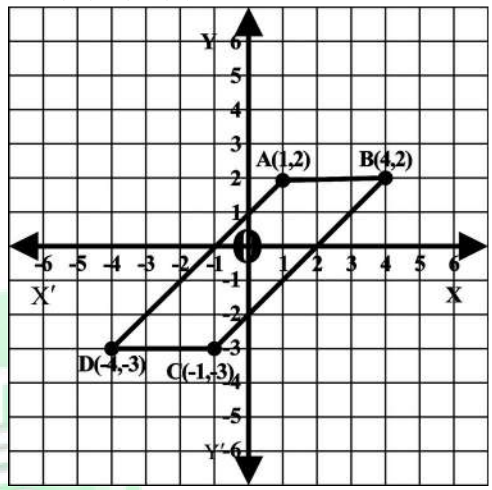
$$M_2(x,y) = M_2\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M_2(x, y) = M_2\left(\frac{4-4}{2}, \frac{2-3}{2}\right)$$

$$M_2(x,y) = M_2\left(\frac{0}{2}, \frac{-1}{2}\right)$$

$$M_2(x,y) = M_2\left(0, \frac{-1}{2}\right)$$

As M_1 and M_2 Coincide. So the diagonals of the parallelogram bisect each other.



Q.6 The vertices of a triangle are $P(4,6), Q(-2,-4)$ and $R(-8,2)$. Show that the length of the line segment joining the midpoints of the line segments $\overline{PR}, \overline{QR}$ is $\frac{1}{2}\overline{PQ}$

(K.B + A.B + U.B)

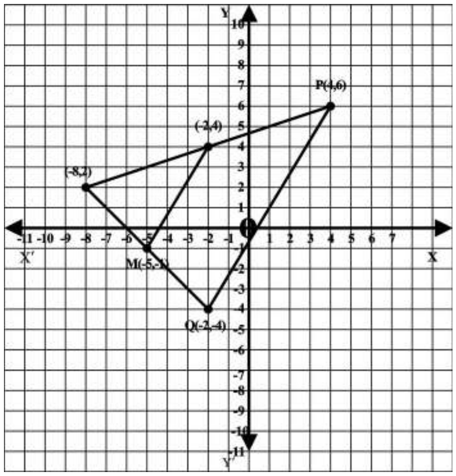
Solution:

M_1 the midpoint of QR is

$$M_1(x,y) = M_1\left(\frac{-2-8}{2}, \frac{-4+2}{2}\right)$$

$$= M_1\left(\frac{-10}{2}, \frac{-2}{2}\right)$$

$$= M_1(-5,-1)$$



M_2 the midpoint of PR is

$$M_2(x,y) = M\left(\frac{4-8}{2},\frac{6+2}{2}\right)$$

$$M_2(x,y) = M_2\left(\frac{-4}{2},\frac{8}{2}\right)$$

$$M_2(x,y) = M_2(-2,4)$$

Now

$$|M_1M_2| = \sqrt{|-5+2|^2 + |4+1|^2}$$

$$|M_1M_2| = \sqrt{(-3)^2 + (5)^2}$$

$$|M_1M_2| = \sqrt{9+25}$$

$$|M_1M_2| = \sqrt{34}$$

$$|P\ Q| = \sqrt{|4+2|^2 + |6+4|^2}$$

$$|P\ Q| = \sqrt{(6)^2 + (10)^2} = \sqrt{36+100}$$

$$|P\ Q| = \sqrt{136}$$

$$|P\ Q| = \sqrt{4\times 34}$$

$$|P\ Q| = 2\sqrt{34}$$

$$\frac{|P\ Q|}{2} = \sqrt{34}$$

OR

$$\frac{1}{2}|PQ| = \sqrt{34}$$

Hence we proved that

$$|M_1M_2| = \frac{1}{2}|PQ|$$