

حل مشق 1.2

سوال 1: درج ذیل مخرج کو ماقط بنائیں۔

(ii) $\frac{\sqrt{2} + \sqrt{5}}{\sqrt{3}}$

$$\begin{aligned} \frac{\sqrt{2} + \sqrt{5}}{\sqrt{3}} &= \frac{\sqrt{2} + \sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\sqrt{3}(\sqrt{2} + \sqrt{5})}{(\sqrt{3})^2} \\ &= \frac{\sqrt{6} + \sqrt{15}}{3} \end{aligned}$$

(iv) $\frac{6 - 4\sqrt{2}}{6 + 4\sqrt{2}}$

$$\begin{aligned} \frac{6 - 4\sqrt{2}}{6 + 4\sqrt{2}} &= \frac{6 - 4\sqrt{2}}{6 + 4\sqrt{2}} \times \frac{6 - 4\sqrt{2}}{6 - 4\sqrt{2}} \\ &= \frac{(6 - 4\sqrt{2})^2}{(6)^2 - (4\sqrt{2})^2} \\ &= \frac{(6)^2 + (4\sqrt{2})^2 - 2(6)(4\sqrt{2})}{36 - 32} \\ &= \frac{36 + 32 - 48\sqrt{2}}{4} = \frac{68 - 48\sqrt{2}}{4} \\ &= \frac{4(17 - 12\sqrt{2})}{4} = 17 - 12\sqrt{2} \end{aligned}$$

(vi) $\frac{4\sqrt{3}}{\sqrt{7} + \sqrt{5}}$

$$\begin{aligned} \frac{4\sqrt{3}}{\sqrt{7} + \sqrt{5}} &= \frac{4\sqrt{3}}{\sqrt{7} + \sqrt{5}} \times \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} - \sqrt{5}} \\ &= \frac{4\sqrt{3}(\sqrt{7} - \sqrt{5})}{(\sqrt{7})^2 - (\sqrt{5})^2} \\ &= \frac{4\sqrt{3}(\sqrt{7} - \sqrt{5})}{7 - 5} \\ &= \frac{4\sqrt{3}(\sqrt{7} - \sqrt{5})}{2} \\ &= 2\sqrt{3}(\sqrt{7} - \sqrt{5}) \end{aligned}$$

(i) $\frac{13}{4 + \sqrt{3}}$

$$\begin{aligned} \frac{13}{4 + \sqrt{3}} &= \frac{13}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}} \\ &= \frac{13(4 - \sqrt{3})}{(4)^2 - (\sqrt{3})^2} = \frac{52 - 13\sqrt{3}}{16 - 3} \\ &= \frac{52 - 13\sqrt{3}}{13} = \frac{52}{13} - \frac{13\sqrt{3}}{13} \\ &= 4 - \sqrt{3} \end{aligned}$$

(iii) $\frac{\sqrt{2} - 1}{\sqrt{5}}$

$$\begin{aligned} \frac{\sqrt{2} - 1}{\sqrt{5}} &= \frac{\sqrt{2} - 1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\sqrt{5}(\sqrt{2} - 1)}{(\sqrt{5})^2} = \frac{\sqrt{10} - \sqrt{5}}{5} \end{aligned}$$

(v) $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

$$\begin{aligned} \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} &= \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} \\ &= \frac{(\sqrt{3} - \sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ &= \frac{(\sqrt{3})^2 + (\sqrt{2})^2 - 2(\sqrt{3})(\sqrt{2})}{3 - 2} \\ &= \frac{3 + 2 - 2\sqrt{6}}{1} = 5 - 2\sqrt{6} \end{aligned}$$

سوال 2: درج ذیل کو حل کریں۔

$$\begin{aligned}
 \text{(ii)} \quad & \left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \frac{16}{27} \\
 & \left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \frac{16}{27} \quad \text{حل:} \\
 & = \left(\frac{4}{3}\right)^2 \div \left(\frac{4}{9}\right)^3 \times \frac{16}{27} \\
 & = \frac{4^2}{3^2} \div \frac{4^3}{9^3} \times \frac{16}{27} = \frac{16}{9} \div \frac{64}{729} \times \frac{16}{27} \\
 & = \frac{16}{9} \times \frac{729}{64} \times \frac{16}{27} = 12
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & \sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^7}} \\
 & \sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^7}} \quad \text{حل:} \\
 & = \left(\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} \times z^7}\right)^{\frac{1}{7}} \\
 & = (x^{14} \times y^{21-14} \times z^{35-7})^{\frac{1}{7}} \\
 & = (x^{14} \times y^7 \times z^{28})^{\frac{1}{7}} \\
 & = x^{14 \times \frac{1}{7}} \times y^{7 \times \frac{1}{7}} \times z^{28 \times \frac{1}{7}} \\
 & = x^2 \times y \times z^4 = x^2 y z^4
 \end{aligned}$$

$$\text{(v)} \quad \frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}}$$

$$\begin{aligned}
 & \frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}} = \frac{5 \cdot (5^2)^{n+1} - 5^2 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (5^2)^{n+1}} \\
 & = \frac{5 \cdot (5^{2n+2}) - 5^{2n+2}}{5^{2n+3+1} - 5^{2n+2}} = \frac{5^{2n+2} (5-1)}{5^{2n+2} (5^2-1)} \\
 & = \frac{4}{25-1} = \frac{4}{24} = \frac{1}{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad & \left(\frac{81}{16}\right)^{-\frac{3}{4}} \\
 & \left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{16}{81}\right)^{\frac{3}{4}} \quad \text{حل:} \\
 & = \frac{(16)^{\frac{3}{4}}}{(81)^{\frac{3}{4}}} = \frac{2^{4 \times \frac{3}{4}}}{3^{4 \times \frac{3}{4}}} \\
 & = \frac{2^3}{3^3} = \frac{8}{27}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & (0.027)^{-\frac{1}{3}} \\
 & (0.027)^{-\frac{1}{3}} = \left(\frac{27}{1000}\right)^{-\frac{1}{3}} \quad \text{حل:} \\
 & = \left(\frac{1000}{27}\right)^{\frac{1}{3}} = \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}} \\
 & = \frac{10^{3 \times \frac{1}{3}}}{3^{3 \times \frac{1}{3}}} = \frac{10}{3}
 \end{aligned}$$

(vi) $\frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$

$$\frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}} = \frac{(2^4)^{x+1} + (5 \times 2^2)(2^2)^{2x}}{2^{x-3} \times (2^3)^{x+2}}$$

حل:

$$= \frac{2^{4(x+1)} + 5 \times 2^2 \times 2^{2 \times 2x}}{2^{x-3} \times 2^{3(x+2)}}$$

$$= \frac{2^{4x+4} + 5 \times 2^{2+4x}}{2^{x-3} \times 2^{3x+6}} = \frac{2^{4x+4} + 5 \times 2^{4x+2}}{2^{x-3+3x+6}}$$

$$= \frac{2^{4x+4} + 5 \times 2^{4x+2}}{2^{4x+3}} = \frac{2^{4x+2} \times 2^2 + 5 \times 2^{4x+2}}{2^{4x+2} \times 2}$$

$$= \frac{2^{4x+2}(2^2 + 5)}{2^{4x+2} \times 2} = \frac{4 + 5}{2} = \frac{9}{2}$$

(vii) $(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

$$(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}} = (4^3)^{-\frac{2}{3}} \div (3^2)^{-\frac{3}{2}}$$

حل:

$$= 4^{3 \times \left(-\frac{2}{3}\right)} \div (3^2)^{-\frac{3}{2}}$$

$$= 4^{-2} \div 3^{-3} = \frac{1}{4^2} \div \frac{1}{3^3}$$

$$= \frac{1}{16} \div \frac{1}{27} = \frac{27}{16}$$

(viii) $\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

$$\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}} = \frac{3^n \times (3^2)^{n+1}}{3^{n-1} \times (3^2)^{n-1}} = \frac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}}$$

حل:

$$= \frac{3^{n+2n+2}}{3^{n-1+2n-2}} = \frac{3^{3n+2}}{3^{3n-3}}$$

$$= \frac{3^{3n} \times 3^2}{3^{3n} \times 3^{-3}} = \frac{3^2}{3^{-3}}$$

$$= 3^{2+3} = 3^5 = 243$$

$$\begin{aligned}
 \text{(ix)} \quad & \frac{5^{n+3} - 6 \cdot 5^{n+1}}{9 \times 5^n - 4 \times 5^n} \\
 & \frac{5^{n+3} - 6 \cdot 5^{n+1}}{9 \times 5^n - 4 \times 5^n} = \frac{5^n \times 5^3 - 6 \cdot 5^n \cdot 5^1}{9 \times 5^n - 4 \times 5^n} \\
 & = \frac{5^n(5^3 - 6 \times 5)}{5^n(9 - 4)} \\
 & = \frac{125 - 30}{5} = \frac{95}{5} = 19
 \end{aligned}$$

حل:

سوال 3: اگر $x = 3 + \sqrt{8}$ ہو تو درج ذیل کی قیمتیں معلوم کریں۔

$$\begin{aligned}
 \text{(i)} \quad & x + \frac{1}{x} \\
 & x = 3 + \sqrt{8} \quad \dots \text{(i)}
 \end{aligned}$$

حل:

$$\begin{aligned}
 \frac{1}{x} &= \frac{1}{3 + \sqrt{8}} \\
 &= \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}} \\
 &= \frac{3 - \sqrt{8}}{(3)^2 - (\sqrt{8})^2} = \frac{3 - \sqrt{8}}{9 - 8} \\
 \frac{1}{x} &= 3 - \sqrt{8} \quad \dots \text{(ii)}
 \end{aligned}$$

مساوات (i) اور (ii) کو جمع کرنے سے

$$\begin{aligned}
 x + \frac{1}{x} &= 3 + \sqrt{8} + 3 - \sqrt{8} \\
 x + \frac{1}{x} &= 6 \quad \dots \text{(iii)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & x - \frac{1}{x} \\
 & x - \frac{1}{x}
 \end{aligned}$$

حل:

مساوات (i) اور (ii) کو تفریق کرنے سے

$$\begin{aligned}
 x - \frac{1}{x} &= 3 + \sqrt{8} - (3 - \sqrt{8}) \\
 &= 3 + \sqrt{8} - 3 + \sqrt{8} \\
 &= 2\sqrt{8}
 \end{aligned}$$

$$(iv) \quad x^2 - \frac{1}{x^2}$$

حل: چونکہ

$$x + \frac{1}{x} = 6$$

$$\left(x - \frac{1}{x}\right) = 2\sqrt{8} \quad \text{اور}$$

ہم جانتے ہیں کہ

$$\begin{aligned} x^2 - \frac{1}{x^2} &= \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) \\ &= (6)(2\sqrt{8}) \\ &= 12\sqrt{8} \end{aligned}$$

$$(vi) \quad \left(x - \frac{1}{x}\right)^2$$

$$x - \frac{1}{x} = 2\sqrt{8} \quad \text{حل: چونکہ}$$

دونوں طرف مربع لینے سے

$$\left(x - \frac{1}{x}\right)^2 = (2\sqrt{8})^2$$

$$\left(x - \frac{1}{x}\right)^2 = 32$$

$$(iii) \quad x^2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2}$$

حل:

مساوات (iii) سے

$$x + \frac{1}{x} = 6$$

دونوں طرف مربع لینے سے

$$\left(x + \frac{1}{x}\right)^2 = (6)^2$$

$$x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 36$$

$$x^2 + \frac{1}{x^2} + 2 = 36$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 34$$

$$(v) \quad x^4 + \frac{1}{x^4}$$

$$x^2 - \frac{1}{x^2} = 12\sqrt{8} \quad \text{حل: چونکہ}$$

دونوں طرف مربع لینے سے

$$\left(x^2 - \frac{1}{x^2}\right)^2 = (12\sqrt{8})^2$$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 2(x^2)\left(\frac{1}{x^2}\right) = 144(8)$$

$$x^4 + \frac{1}{x^4} - 2 = 1152$$

$$x^4 + \frac{1}{x^4} = 1154$$

سوال 4: ناظم اعداد p اور q معلوم کریں اس طرح کہ $\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$$

.... (i)

حل:

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = \frac{8-3\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}}$$

$$= \frac{(8-3\sqrt{2})(4-3\sqrt{2})}{(4)^2 - (3\sqrt{2})^2} = \frac{8(4-3\sqrt{2}) - 3\sqrt{2}(4-3\sqrt{2})}{16 - 9(2)}$$

$$\begin{aligned}
&= \frac{32 - 24\sqrt{2} - 12\sqrt{2} + 9(\sqrt{2})^2}{16 - 18} \\
&= \frac{32 - 36\sqrt{2} + 18}{-2} = \frac{50 - 36\sqrt{2}}{-2} \\
&= \frac{50}{-2} - \frac{36\sqrt{2}}{(-2)} = -25 + 18\sqrt{2}
\end{aligned}$$

لہذا مساوات (i) ہوگی۔

$$\Rightarrow -25 + 18\sqrt{2} = p + q\sqrt{2}$$

p اور q کے عددی سروں کا موازنہ کرنے سے

$$p = -25, q = 18$$

سوال 5: درج ذیل کو مختصر کریں۔

$$\begin{aligned}
\text{(i)} \quad & \frac{(25)^{\frac{3}{2}} \times (243)^{\frac{5}{4}}}{(16)^{\frac{4}{3}} \times (8)^{\frac{3}{5}}} \\
&= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{5}{4}}}{(2^4)^{\frac{4}{3}} \times (2^3)^{\frac{3}{5}}} \\
&= \frac{5^{2 \times \frac{3}{2}} \times 3^{5 \times \frac{5}{4}}}{2^{4 \times \frac{4}{3}} \times 2^{3 \times \frac{3}{5}}} \\
&= \frac{5^3 \times 3^3}{2^5 \times 2^4} = \frac{125 \times 27}{32 \times 16} = \frac{3375}{512}
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad & \frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})} \\
&= \frac{2 \times 3 \times 3 \times 3 \times ((3 \times 3 \times 3)^{\frac{2x}{3}})^{\frac{1}{3}}}{(3 \times 3)^{x+1} + (2 \times 2 \times 2 \times 3 \times 3 \times 3)(3)^{2x-1}} = \frac{2^1 \times 3^3 \times ((3^3)^{\frac{2x}{3}})^{\frac{1}{3}}}{(3^2)^{x+1} + (2^3 \times 3^3)(3)^{2x-1}} \\
&= \frac{2 \times 3^3 \times (3^{6x})^{\frac{1}{3}}}{3^{2(x+1)} + 2^3 \times 3^3 \times 3^{2x-1}} = \frac{2 \times 3^3 \times 3^{2x}}{3^{2x+2} + 2^3 \times 3^{3+2x-1}} \\
&= \frac{2 \times 3^{2x+3}}{3^{2x+2} + 2^3 \times 3^{2x+2}} = \frac{3^{2x+2+1} \times 2}{3^{2x+2}(1+2^3)} = \frac{3^{2x+2} \times 3^1 \times 2}{3^{2x+2}(1+8)} \\
&= \frac{3 \times 2}{9} = \frac{6}{9} = \frac{2}{3}
\end{aligned}$$

$$(iii) \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$$

$$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}} = \sqrt{\frac{(2^3 \times 3^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{-3}{2}}}}$$

حل:

$$= \sqrt{\frac{2^{3 \times \frac{2}{3}} \times 3^{3 \times \frac{2}{3}} \times 5^{2 \times \frac{1}{2}}}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}} = \sqrt{\frac{2^2 \times 3^2 \times 5}{(25)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{2^2 \times 3^2 \times 5}{5^{2 \times \frac{3}{2}}}} = \left(\frac{2^2 \times 3^2 \times 5}{5^3}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^2 \times 3^2}{5^{3-1}}\right)^{\frac{1}{2}} = \frac{2^{2 \times \frac{1}{2}} \times 3^{2 \times \frac{1}{2}}}{5^{2 \times \frac{1}{2}}}$$

$$= \frac{2 \times 3}{5} = \frac{6}{5}$$

$$(iv) \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$$

$$\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$$

حل:

$$= a^{\frac{1}{3}} \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right) + b^{\frac{2}{3}} \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$$

$$= \left(a^{\frac{2}{3} + \frac{1}{3}} - a^{\frac{1}{3} + \frac{1}{3}}b^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{4}{3}}\right) + \left(a^{\frac{2}{3}}b^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3} + \frac{2}{3}} + b^{\frac{4}{3} + \frac{2}{3}}\right)$$

$$= a^{\frac{3}{3}} - a^{\frac{2}{3}}b^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{4}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{4}{3}} + b^{\frac{6}{3}}$$

$$= a - \cancel{a^{\frac{2}{3}}b^{\frac{2}{3}}} + \cancel{a^{\frac{1}{3}}b^{\frac{4}{3}}} + \cancel{a^{\frac{2}{3}}b^{\frac{2}{3}}} - \cancel{a^{\frac{1}{3}}b^{\frac{4}{3}}} + b^2$$

$$= a + b^2$$