

EXERCISE 1.2

Q.1 Rationalize the denominator of following:

(i) $\frac{13}{4+\sqrt{3}}$

Ans:

Rationalize the denominator

$$\begin{aligned} &= \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} \\ &= \frac{13(4-\sqrt{3})}{(4)^2 - (\sqrt{3})^2} = \frac{13(4-\sqrt{3})}{16-3} \\ &= \frac{13(4-\sqrt{3})}{13} = 4-\sqrt{3} \end{aligned}$$

(ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

Ans:

Rationalize the denominator

$$\begin{aligned} &= \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\sqrt{3}(\sqrt{2}+\sqrt{5})}{(\sqrt{3})^2} \Rightarrow \frac{\sqrt{3 \times 2} + \sqrt{5 \times 3}}{3} \\ &= \frac{\sqrt{6} + \sqrt{15}}{3} \end{aligned}$$

(iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Ans:

Rationalize the denominator

$$\begin{aligned} &= \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\sqrt{5}(\sqrt{2}-1)}{(\sqrt{5})^2} \\ &= \frac{\sqrt{5 \times 2} - \sqrt{5}}{5} \\ &= \frac{\sqrt{10} - \sqrt{5}}{5} \end{aligned}$$

(iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Ans:

Rationalize the denominator

$$\begin{aligned} &= \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}} \\ &= \frac{(6-4\sqrt{2})^2}{(6)^2 - (4\sqrt{2})^2} \\ &= \frac{(6)^2 + (4\sqrt{2})^2 - 2(6)(4)\sqrt{2}}{36 - 16 \times 2} \\ &= \frac{36 + 16 \times 2 - 48\sqrt{2}}{36 - 32} \\ &= \frac{36 + 32 - 48\sqrt{2}}{4} \\ &= \frac{68 - 48\sqrt{2}}{4} \\ &= \frac{4[17 - 12\sqrt{2}]}{4} \\ &= 17 - 12\sqrt{2} \end{aligned}$$

(v) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Ans:

Rationalize the denominator

$$\begin{aligned} &= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \\ &= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ &= \frac{(\sqrt{3})^2 + (\sqrt{2})^2 - 2(\sqrt{3})(\sqrt{2})}{3-2} \\ &= \frac{3+2-2\sqrt{3 \times 2}}{1} \\ &= \sqrt{5} - 2\sqrt{6} \end{aligned}$$

(vi) $\frac{4\sqrt{3}}{\sqrt{7} + \sqrt{5}}$

Ans:

$$\begin{aligned} & \frac{4\sqrt{3}}{\sqrt{7} + \sqrt{5}} \times \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} - \sqrt{5}} \\ &= \frac{4\sqrt{3}(\sqrt{7} - \sqrt{5})}{(\sqrt{7})^2 - (\sqrt{5})^2} \\ &= \frac{4\sqrt{3}(\sqrt{7} - \sqrt{5})}{7 - 5} \\ &= \frac{4\sqrt{3}(\sqrt{7} - \sqrt{5})}{2} \\ &= 2(\sqrt{21} - \sqrt{15}) \end{aligned}$$

Q.2 Simplify the following

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

Ans:

$$\begin{aligned} &= \left(\frac{16}{81}\right)^{\frac{3}{4}} = \left[\frac{2^4}{3^4}\right]^{\frac{3}{4}} \\ &= \frac{2^{4 \times \frac{3}{4}}}{3^{4 \times \frac{3}{4}}} = \frac{2^3}{3^3} = \frac{8}{27} \end{aligned}$$

(ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right) \times \frac{16}{27}$

Ans:

$$\begin{aligned} & \left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right) \times \frac{16}{27} = \left(\frac{4}{3}\right)^2 \div \frac{4}{9^3} \times \frac{16}{27} \\ &= \frac{4^2}{3^2} \times \frac{(3^2)^3}{4^3} \times \frac{4^2}{3^3} = \frac{4^2}{3^2} \times \frac{3^6}{4^3} \times \frac{4^2}{3^3} \\ &= 4^2 \times 4^{-3} \times 4^2 \times 3^6 \times 3^{-3} \times 3^{-2} \\ &= 4^1 \times 3^1 = 12 \end{aligned}$$

(iii) $(0.027)^{-\frac{1}{3}}$

Ans:

$$\begin{aligned} &= (27 \times 10^{-3})^{-\frac{1}{3}} = (3^3 \times 10^{-3})^{-\frac{1}{3}} \\ &= 3^{3 \times -\frac{1}{3}} \times 10^{-3 \times -\frac{1}{3}} = 3^{-1} \times 10^1 \\ &= \frac{10}{3} \end{aligned}$$

(iv) $\sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^7}}$

Ans:

$$\begin{aligned} &= \left[x^{14} \times y^{421} \times y^{-14} \times z^{35} \times z^{-7} \right]^{\frac{1}{7}} \\ &= \left[x^{14} \times y^7 \times z^{28} \right]^{\frac{1}{7}} = x^{14 \times \frac{1}{7}} y^{7 \times \frac{1}{7}} z^{28 \times \frac{1}{7}} \\ &= x^2 y z^4 \end{aligned}$$

(v) $\frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}}$

Ans:

$$\begin{aligned} &= \frac{5^1 \cdot (5^2)^{n+1} - 5^{-2} \cdot (5)^{2n}}{5^1 \cdot 5^{2n+3} - (5^2)^{n+1}} \\ &= \frac{5^1 (5)^{2n+2} - 5^{2n+2}}{5^{2n+4} - 5^{2n+2}} = \frac{5^{2n+2} 5 - 5^{2n+2}}{5^{2n+2} \times 5^2 - 5^{2n+2}} \\ &= \frac{5^{2n+2} (5-1)}{5^{2n+2} [5^2 - 1]} = \frac{4}{25-1} \\ &= \frac{4}{24} = \frac{1}{6} \end{aligned}$$

(vi) $\frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$

Ans:

$$\begin{aligned} &= \frac{(4^2)^{x+1} + 20(4^{2x})}{2^{x-3} \times (2^3)^{x+2}} \\ &= \frac{4^{2x+2} + 20(4)^{2x}}{2^{x-3} \times 2^{3x+6}} \\ &= \frac{4^{2x} \times 4^2 + 20(4)^{2x}}{2^{x-3+3x+6}} \\ &= \frac{4^{2x} (4^2 + 20)}{2^{4x+3}} = \frac{(2^2)^{2x} (16+20)}{2^{4x} \times 2^3} \\ &= \frac{2^{4x} \times (36)}{2^{4x} \times 8} = \frac{36}{8} = \frac{9}{2} \end{aligned}$$

(vii) $(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

Ans:

$$\begin{aligned} &= \frac{(4^3)^{-\frac{2}{3}}}{(3^2)^{-\frac{3}{2}}} = \frac{4^{3 \times -\frac{2}{3}}}{3^{2 \times -\frac{3}{2}}} \\ &= \frac{4^{-2}}{3^{-3}} = \frac{3^3}{4^2} = \frac{27}{16} \end{aligned}$$

(viii) $\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

Ans:

$$\begin{aligned} &= 3^n \times 3^{-n+1} \times 9^{n+1} \times 9^{-n-1} \\ &= 3^{n-n+1} \times 9^{n+1-n-1} \\ &= 3^1 \times 9^0 = 3 \times 1 = 3 \end{aligned}$$

(ix) $\frac{5^{n+3} - 6 \cdot 5^{n+1}}{7 \times 5^n - 2 \times 5^n}$

Ans:

$$\begin{aligned} \frac{5^{n+3} - 6 \cdot 5^{n+1}}{7 \times 5^n - 2 \cdot 5^n} &= \frac{5^n \cdot 5^3 - 6 \cdot 5^n \times 5}{5^n (7 - 2)} \\ &= \frac{5^n (5^3 - 30)}{5^n (5)} = \frac{125 - 30}{5} = \frac{95}{5} = 19 \end{aligned}$$

Q.3 If $x = 3 + \sqrt{8}$ then find the value of:

Ans:

$$x = 3 + \sqrt{8}$$

$$\frac{1}{x} = \frac{1}{3 + \sqrt{8}}$$

To rationalize multiply and divide by $3 - \sqrt{8}$

$$\frac{1}{x} = \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$$

$$\frac{1}{x} = \frac{3 - \sqrt{8}}{(3)^2 - (\sqrt{8})^2}$$

$$\frac{1}{x} = \frac{3 - \sqrt{8}}{9 - 8} = \frac{3 - \sqrt{8}}{1}$$

$$\frac{1}{x} = 3 - \sqrt{8}$$

(i) $x + \frac{1}{x}$

Ans:

$$x + \frac{1}{x} = 3 + \sqrt{8} + 3 - \sqrt{8}$$

$$x + \frac{1}{x} = 6$$

(ii) $x - \frac{1}{x}$

Ans:

$$x - \frac{1}{x} = 3 + \sqrt{8} - [3 - \sqrt{8}]$$

$$x - \frac{1}{x} = 3 + \sqrt{8} - 3 + \sqrt{8}$$

$$x - \frac{1}{x} = 2\sqrt{8}$$

(iii) $x^2 + \frac{1}{x^2}$

Ans:

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$(6)^2 = x^2 + \frac{1}{x^2} + 2$$

$$36 = x^2 + \frac{1}{x^2} + 2$$

$$36 - 2 = x^2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2} = 34$$

(iv) $x^2 - \frac{1}{x^2}$

Ans:

$$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$= (6)(2\sqrt{8}) = 12\sqrt{8}$$

(v) $x^4 + \frac{1}{x^4}$

Ans:

$$x^2 - \frac{1}{x^2} = 34$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (34)^2$$

$$x^4 + \frac{1}{x^4} + 2 = 1156$$

$$x^4 + \frac{1}{x^4} = 1156 - 2$$

$$x^4 + \frac{1}{x^4} = 1154$$

(vi) $\left(x - \frac{1}{x}\right)^2$

Ans:

$$\left(x - \frac{1}{x}\right)^2 = (2\sqrt{8})^2 = 4 \times 8$$

$$\left(x - \frac{1}{x}\right)^2 = 32$$

Q.4 Find the rational numbers p and q

such that $\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$

Ans:

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = p + q\sqrt{2}$$

$$\frac{32-24\sqrt{2}-12\sqrt{2}+9\sqrt{2} \times 2}{(4)^2-(3\sqrt{2})^2} = p + q\sqrt{2}$$

$$\frac{32-36\sqrt{2}+9\sqrt{4}}{16-9 \times 2} = p + q\sqrt{2}$$

$$\frac{32+3 \times 2-36\sqrt{2}}{16-18} = p + q\sqrt{2}$$

$$\frac{32+6-36\sqrt{2}}{-2} = p + q\sqrt{2}$$

$$\frac{38-36\sqrt{2}}{-2} = p + q\sqrt{2}$$

$$\frac{2(19-18\sqrt{2})}{-2} = p + q\sqrt{2}$$

$$-19-18\sqrt{2} = p + q\sqrt{2}$$

$$-19 = p \Rightarrow p = -19$$

$$18\sqrt{2} = q\sqrt{2}$$

$$\frac{18\sqrt{2}}{\sqrt{2}} = q$$

$$q = 18$$

Q.5 Simplify the following

(i) $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

Ans:

$$= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} (2^3)^{\frac{4}{3}}} = \frac{5^{2 \times \frac{3}{2}} 3^{5 \times \frac{3}{5}}}{2^{4 \times \frac{5}{4}} \times 2^{3 \times \frac{4}{3}}}$$

$$= \frac{5^3 \times 3^3}{2^5 \times 2^4} = \frac{125 \times 27}{32 \times 16} = \frac{3375}{512}$$

(ii) $\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

Ans:

$$\begin{aligned} &= \frac{2 \times 3^3 \times \left[(3^3)^{2x}\right]^{\frac{1}{3}}}{(3^2)^{x+1} + 2^3 \times 3^3 \times 3^{2x-1}} \\ &= \frac{2 \times 3^3 \times 3^{2x}}{3^{2x+2} + 2^3 \times 3^{2x+2}} = \frac{2 \times 3 \times 3 \times 3 \times 3^{2x}}{3^{2x+2} [1+2^3]} \\ &= \frac{2 \times 3 \times 3^{2x+2}}{3^{2x+2} (1+8)} = \frac{6}{9} = \frac{2}{3} \end{aligned}$$

(iii) $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

Ans:

$$\begin{aligned} &= \sqrt{\left(6^3\right)^{\frac{2}{3}} \left(5\right)^{\frac{1}{2}} (0.04)^{\frac{3}{2}}} \\ &= \sqrt{6^{3 \times \frac{2}{3}} \times 5^{2 \times \frac{1}{2}} \times \left(\frac{4}{100}\right)^{\frac{3}{2}}} \\ &= \sqrt{6^2 \times 5 \times \left(\frac{1}{5^2}\right)^{\frac{3}{2}}} = \sqrt{6^2 \times 5 \times \frac{1}{5^{2 \times \frac{3}{2}}}} \\ &= \sqrt{6^2 \times 5 \times \frac{1}{5^3}} = \sqrt{6^2 \times 5 \times \frac{1}{5 \times 5^2}} \\ &= \sqrt{\frac{6^2}{5^2}} = \sqrt{\left(\frac{6}{5}\right)^2} = \frac{6}{5} \end{aligned}$$

(iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

Ans:

$$= \left[a^{\frac{1}{3}} + b^{\frac{2}{3}}\right] \left[\left(a^{\frac{1}{3}}\right)^2 - a^{\frac{1}{3}}b^{\frac{2}{3}} + \left(b^{\frac{2}{3}}\right)^2\right]$$

Formula $(a+b)(a^2-ab+b^2) = a^3+b^3$

$$\begin{aligned} &= \left(a^{\frac{1}{3}}\right)^3 + \left(b^{\frac{2}{3}}\right)^3 = a^{\frac{1}{3} \times 3} + b^{\frac{2}{3} \times 3} \\ &= a + b^2 \end{aligned}$$