

EXERCISE 4.2

- Factorize each of the following expressions:

- $4x^4 + 81y^4$

Ans:

$$= (2x^2)^2 + (9y^2)^2$$

Adding and subtracting by $36x^2y^2$

$$(2x^2)^2 + (9y^2)^2 + 36x^2y^2 - 36x^2y^2$$

$$= (2x^2 + 9y^2)^2 - 36x^2y^2$$

$$= (2x^2 + 9y^2)^2 - (6xy)^2$$

$$= [2x^2 + 9y^2 - 6xy][2x^2 + 9y^2 + 6xy]$$

- $a^4 + 64b^4$

Ans:

$$= (a^2)^2 + (8b^2)^2$$

Adding and subtracting by $16a^2b^2$

$$= (a^2)^2 + (8b^2)^2 + 16a^2b^2 - 16a^2b^2$$

$$= (a^2 + 8b^2)^2 - 16a^2b^2$$

$$= (a^2 + 8b^2)^2 - (4ab)^2$$

$$= [a^2 + 8b^2 - 4ab][a^2 + 8b^2 + 4ab]$$

- $x^4 + 4x^2 + 16$

Ans:

$$= (x^2)^2 + (4)^2 + 4x^2$$

Adding and subtracting by $8x^2$

$$= (x^2)^2 + (4)^2 + 8x^2 - 8x^2 + 4x^2$$

$$(x^2 + 4^2) - 6x^2$$

$$6x^2 = \sqrt{6}x$$

$$\text{So, } = (x^2 + 4)^2 - (\sqrt{6}x)^2$$

$$= [x^2 + 4 - \sqrt{6}x][x^2 + 4 + \sqrt{6}x]$$

- $x^4 - 14x^2 + 1$

Ans:

$$= (x^2)^2 + (1)^2 - 14x^2$$

Adding and subtracting by $2x^2$

$$= (x^2)^2 + (1)^2 + 2x^2 - 2x^2 - 14x^2$$

$$= (x^2 + 1)^2 - 16x^2$$

$$= (x^2 + 1)^2 - (4x)^2$$

$$= [x^2 + 1 - 4x][x^2 + 1 + 4x]$$

$$= [x^2 - 4x + 1][x^2 + 4x + 1]$$

- $x^4 - 30x^2y^2 + 9y^4$

Ans:

$$= (x^2)^2 + (3y^2)^2 + 6x^2y^2 - 6x^2y^2 - 30x^2y^2$$

$$= (x^2 + 3y^2)^2 - 36x^2y^2$$

$$= (x^2 + 3y^2)^2 - (6xy)^2$$

$$= (x^2 + 3y^2 - 6xy)(x^2 + 3y^2 + 6xy)$$

- $x^4 + 11x^2y^2 + y^4$

Ans:

$$x^4 - 7x^2y^2 + y^4$$

$$x^4 + y^4 - 7x^2y^2$$

$$(x^2)^2 + (y^2)^2 - 7x^2y^2$$

Adding and subtracting by $2x^2y^2$

$$(x^2)^2 + (y^2)^2 + 2x^2y^2 - 2x^2y^2 - 7x^2y^2$$

$$(x^2 + y^2)^2 - 9x^2y^2$$

$$(x^2 + y^2)^2 - (3xy)^2$$

$$[x^2 + y^2 - 3xy][x^2 + y^2 + 3xy]$$

$$= (x^2 + 5x + 5)$$

- Factorize each of the following expressions:**

- $(x+1)(x+2)(x+3)(x+4)+1$

Ans:

$$= [x+1][x+4][x+2][x+3]+1$$

$$= [x^2 + 4x + 1x + 4][x^2 + 3x + 2x + 6] + 1$$

$$= [x^2 + 5x + 4][x^2 + 5x + 6] + 1$$

Let $x^2 + 5x = y$

$$= [y+4][y+6]+1$$

$$= y^2 + 6y + 4y + 24 + 1$$

$$= y^2 + 10y + 25$$

$$= y^2 + 5y + 5y + 25$$

$$= y(y+5)(y+5)$$

As we know that $y = x^2 + 5x$

$$= [x^2 + 5x + 5][x^2 + 5x + 5]$$

- $(x+2)(x-7)(x-4)(x-1)+17$

Ans:

$$= [x^2 - 7x + 2x - 14][x^2 - x - 4x + 4] + 17$$

$$= [x^2 - 5x - 14][x^2 - 5x + 4] + 17$$

Let $x^2 - 5x = y$

$$= [y-14][y+4]+17$$

$$= y^2 + 4y - 14y - 56 + 17$$

$$= y^2 - 10y - 39$$

$$= y(y-13)+3(y-13)$$

$$= (y-13)(y+3)$$

As we know $x^2 - 5x = y$

$$(x^2 - 5x - 13)(x^2 - 5x + 3)$$

- $(2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1$

Ans:

Let $2x^2 + 7x = 7$

$$= (y+3)(y+5)+1$$

$$= y^2 + 5y + 3y + 15 + 1$$

$$= y^2 + 8y + 16$$

$$= y^2 + 4y + 4y + 16$$

$$= y(y+4)+4(y+4)$$

$$= (y+4)(y+4)$$

As we know $2x^2 + 7x = y$

$$= (2x^2 + 7x + 4)(2x^2 + 7x + 4) = (2x^2 + 7x + 4)^2$$

- $(3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3$

Ans:

$$3x^2 + 5x = y$$

$$= (y+3)(y+5)-3$$

$$= y^2 + 5y + 3y + 15 - 3$$

$$= y^2 + 5y + 3y + 15 - 3$$

$$= y^2 + 8y + 12$$

$$= y^2 + 1y + 2y + 12$$

$$= y(y+6)+2(y+6)$$

$$= (y+6)(y+2)$$

We know that $y = 3x^2 + 5x$

$$(3x^2 + 5x + 6)(3x^2 + 5x + 2)$$

- $(x+1)(x+2)(x+3)(x+6)-3x^2$

Ans:

$$= [x+1][x+6][x+2][x+3]-3x^2$$

$$= [x^2 + 6x + x + 6][x^2 + 3x + 2x + 6] - 3x^2$$

$$= [x^2 + 7x + 6][x^2 + 5x + 6] - 3x^2$$

$$= [x^2 + 6 + 7x][x^2 + 6 + 5x] - 3x^2$$

Let $x^2 + 6 = y$

$$= [y+7x][y+5x]-3x^2$$

$$= y^2 + 5xy + 7xy + 35x^2 - 3x^2$$

$$= y^2 + 12xy + 32x^2$$

$$= y^2 + 8xy + 4xy + 32x^2$$

$$= y(y + 8x) + 4x(y + 8x)$$

$$= (y + 8x)(y + 4x)$$

$$\text{As we know } y = x^2 + 6$$

$$= [x^2 + 6 + 8x][x^2 + 6 + 4x]$$

$$= [x^2 + 8x + 6][x^2 + 4x + 6]$$

- $(x+1)(x-1)(x+2)(x-2) + 5x^2$

Ans:

$$(x+1)(x-1)(x+2)(x-2) + 5x^2$$

$$= (x+1)(x+2)(x-1)(x-2) + 5x^2$$

$$= [x^2 + 2x + x + 2][x^2 - 2x - x + 2] + 5x^2$$

$$= [x^2 + 3x + 2][x^2 - 3x + 2] + 5x^2$$

$$= [x^2 + 2 + 3x][x^2 + 2 - 3x] + 5x^2$$

$$\text{Let } x^2 + 2 = y$$

$$= (y + 3x)(y - 3x) + 5x^2$$

$$y^2 - 3xy + 3xy - 9x^2 + 5x^2$$

$$= y^2 - 4x^2$$

$$= y^2 - (2x)^2$$

$$= (y - 2x)(y + 2x)$$

$$= (x^2 + 2 - 2x)(x^2 + 2 + 2x)$$

$$= (x^2 - 2x + 2)(x^2 + 2x + 2)$$

- **Factorize:**

- $8x^3 + 12x^2 + 6x + 1$

Ans:

By using formula

$$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$= (2x)^3 + (1)^3 + 3(2x)(1) + 3(2x)(1)$$

$$= [2x+1]^3$$

- $27a^3 + 108a^2b + 144ab^2 + 64b^3$

Ans:

- $x^3 + 48x^2y + 108xy^2 + 216y^3$

Ans:

$$= (x)^3 + (6y)^3 + 3(x)^2(6y) + 3(x)(6y)^2$$

$$= (x+6y)^3$$

- $8x^3 - 125y^3 + 150xy^2 - 60x^2y$

Ans:

$$= (2x)^3 - (5y)^3 + 3(2x)^2(5y) - 3(2x)(5y)^2$$

$$= [2x-5y]^3$$

- **Factorize:**

- $125a^3 - 1$

Ans:

Using formula

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$= (5a)^3 - (1)^3$$

$$= [5a-1][(5a)^2 + (5a)(1) + (1)^2]$$

$$= [5a-1][25a^2 + 5a + 1]$$

- $64x^3 + 125$

Ans:

Using formula

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$= (4x)^3 + (5)^3$$

$$= (4x+5)[(4x)^2 - (4x)(5) + (5)^2]$$

$$= [4x+5][16x^2 - 20x + 25]$$

- $x^6 - 27$

Ans:

By using formula

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$= (x^2)^3 - 3^3$$

$$= (x^2-3)[(x^2)^2 + (x^2)(3) + (3)^2]$$

$$= (x^2-3)(x^4 + 3x^2 + 9)$$

- $1000a^3 + 1$

Ans:

By using formula

$$\begin{aligned} & (10a)^3 + (1)^3 \\ &= [10a + 1] \left[(10a)^2 - (10a)(1) + (1)^2 \right] \\ &= (10a + 1)(100a^2 - 10a + 1) \end{aligned}$$

- $343x^3 + 216$

Ans:

By using formula

$$\begin{aligned} & a^3 + b^3 = (a + b)(a^2 - ab + b^2) \\ &= (7x)^3 + (6)^3 \\ &= [7x + 6] \left[(7x)^2 - (7x)(6) + (6)^2 \right] \\ &= (7x + 6)[49x^2 - 42x + 36] \end{aligned}$$

- $27 - 512y^3$

Ans:

By using formula

$$\begin{aligned} & a^3 - b^3 = (a - b)(a^2 + ab + b^2) \\ &= (3)^3 - (8y)^3 \\ &= (3 - 8y) \left[(3)^2 + (3)(8y) + (8y)^2 \right] \\ &= (3 - 8y)(9 + 24y + 64y^2) \end{aligned}$$

