EXERCISE 4.3

- **Find HCF by factorization method.**

1. $21x^2y, 35xy^2$

Ans:

$21x^2y = 3\times \underline{7}\times \underline{x}\times x\times \underline{y}$

$35xy^2 = 5\times \underline{7}\times \underline{xy}\times y$

$\text{Common} = 7\times x\times y$

$\text{H.C.F} = 7xy$

2. $4x^2-9y^2, 2x^2-3xy$

Ans:

$4x^2-9y^2 = (2x)^2-(3y)^2$

$=[2x-3y][2x+3y]$

$2x^2-3xy = x(2x-3y)$

$\text{Common} = (2x-3y)$

$\text{H.C.F} = 2x-3y$

3. x^3-1, x^2+x+1

Ans:

$x^3-1 = x^3-(1)^3$

$=(x-1)(x^2+x+1)$

$x^2+x+1 = x^2+x+1$

$\text{Common} = x^2+x+1$

$\text{H.C.F} = x^2+x+1$

4. $a^3+2a^2-3a, 2a^3+5a^2-3a$

Ans:

a^3+2a^2-3a

$=a(a^2+2a-3)$

$=[a^2+3a-a-3]$

$=a(a+3)-1(a+3)$

$=(a+3)(a-1)$

$2a^3+5a^2-3a$

$=a(2a^2+5a-3)$

$=a[2a^2+6a-1a-3]$

$=a[a(a+3)-1(a+3)]$

$=a(a+3)(2a-1)$

$=\text{H.C.F}\ a(a+3)$

5. $t^2+3t-4, t^2+5t+4, t^2-1$

Ans:

$t^2-3t-4=t^2-4t+1t-4$

$=t(t-4)+1(t-4)=(t-4)(t+1)$

$t^2+5t+4=t^2+4t+1t+4$

$=t(t+4)+1(t+4)=(t+4)(t+1)$

$t^2-1=t^2-1^2=(t+1)(t-1)$

H.C.F $(t+1)$

6. $x^2+15x+56, x^2+5x-24, x^2+8x$

Ans:

$x^2+15x+56=x^2+8x+7x+56$

$=x(x+8)+7(x+8)$

$=(x+8)(x+7)$

$x^2+5x-24=x^2+8x-3x-24$

$=x(x+8)-3(x+8)$

$=(x+8)(x-3)$

$x^2+8x=x(x+8)$

Common $=(x+8)$

H.C.F $=x+8$

- **Find HCF of the following expression by using division method.**
- $27x^3+9x^2-3x-10, 3x-2$

Ans:

$9x^2+9x+5$

$3x-2$ | $27x^3+9x-3x-10$

$\pm 27x^3 \mp 18x^2$

$27x^2-3x-10$

$\pm 27x^2 \mp 18x$

$15x-10$

$\pm 15x \mp 10$

H.C.F $=(3x-2)$

- $x^3-9x^2+21x-15, x^2-4x+3$

Ans:

$x-5$

x^2-4x+3 | $x^3-9x^2+21x-9$

$\pm x^3 \mp 4x^2 \pm 3x$

$-5x^2+18x-9$

$\mp 5x^2 \pm 20x \mp 15$

$-2x+6$

$-2(x-3)$

$x-1$

$x-3$ | x^2-4x+3

$\pm x^2 \mp 3x$

$-x+3$

$\mp x \pm 3$

H.C.F $=(x-3)$

- $2x^3+2x^2+2x+2, 6x^3+12x^2+6x+12$

Ans:

3

$2x^3+2x^2+2x+2$ | $6x^3+12x^2 6x+12$

$\pm 6x^3 \pm 6x^2 \pm 6$

$6x^2+6$

$3(2x^2+2)$

$x+1$

$2x^2+2$ | $2x^3+2x^2+2x+2$

$\pm 2x^3 \pm 2x$

$2x^2+2$

$\pm 2x^2 \pm 2$

H.C.F $=2(x^2+1)$

- $2x^3-4x^2+6x, x^3-2x, 3x^2-6x$

Ans:

$x+2$

$3x^2-6x$ | x^3-2x

3

$\pm 3x^3 \mp 6x^2$

$\pm 6x^2 \mp 12x$

$6x$

Since is not the factory polynomial so

are ignore

$$\begin{array}{r|l} & 3x-6 \\ \hline x & 3x^2-6x \\ & \pm 3x^2 \\ \hline & -6x \\ & \pm 6x \\ \hline \end{array}$$

$$\begin{array}{r|l} & 2x^2-4x+6 \\ \hline x & 2x^3-4x+6x \\ & \pm 2x^3 \\ \hline & -4x^2+6x \\ & \pm 6x \\ \hline \end{array}$$

H.C.F = x

- **Find LCM of the following expressions by using prime factorization method.**

1. $2a^2b, 4ab^2, 6ab$

Ans:

$2a^2b = \underline{2} \times \underline{a} \times a \times \underline{b}$

$4ab = \underline{2} \times 2 \times \underline{a} \times \underline{b}$

$6ab = \underline{2} \times 3 \times \underline{a} \times \underline{b}$

Common factor $= 2 \times a \times b = 2ab$

Non common factor $= a \times 2 \times 3 = 6a$

L.C.M = C.F × N.C.F

$= 12a^2b$

2. $x^2 + x, x^3 + x^2$

Ans:

$x^2 + x = x(x+1)$

$x^3 + x^2 = x^2(x+1)$

$= x \times x(x+1)$

C.F $= x(x+1)$

N.C.F $= x$

L.C.M = C.F × N.C.F

$= x(x+1) \times x = x^2(x+1)$

3. $a^2 - 4a + 4, a^2 - 2a$

Ans:

$a^2 - 4a + 4 = a^2 - 2a - 2a + 4$

$= a(a-2) - 2(a-2)$

$= (a-2)(a-2)$

$a^2 - 2a = a(a-2)$

C.F $= (a-2)$

N.C.F $= (a-2)a$

L.C.M = C.F × N.C.F

L.C.M = $(a-2)(a-2)a$

L.C.M = $a(a-2)^2$

4. $x^4 - 16, x^3 - 4x$

Ans:

$x^4 - 16 = (x^2)^2 - 4$

$= (x^2-4)(x^2+4)$

$= (x-2)(x+2)(x^2+4)$

$x^3 - 4x = x(x^2-4)$

$= x(x^2-2^2)$

$= x(x-2)(x+2)$

$C.F = (x-2)(x+2)$

$N.C.F = x(x^2+4)$

$L.C.M = C.F \times N.C.F$

$= (x-2)(x+2)x(x^2+4)$

$= x(x^2-2^2)(x^2+4)$

$= x(x^2-4)$

$= x(x^2-4)(x^2+4)$

$= x(x^4-16)$

5. $16 - 4x^2, x^2 + x - 6, 4 - x^2$

Ans:

$16 - 4x^2 = -4x^2 + 16$

$= -4[x^2-4] = -4[x^2-2^2]$

$= -4[x-2][x+2]$

$x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= x(x+3) - 2(x+3) = (x+3)(x-2)$

$4 - x^2 = -x^2 + 4 = -1(x^2-4)$

$= -1(x^2-2^2) = -1(x-2)(x+2)$

C.F $= (x-2)$

N.C.F

$= -4 \times -1(x+2)(x+3) = 4(x+2)(x+3)$

L.C.M = C.F × N.C.F

$= (x-2)[4(x+2)(x+3)]$

$= 4(x-2)(x+2)(x+3)$

$= 4[x^2-4][x+3]$

- **The HCF of two polynomials is $y-7$ and their LCM is $y^3-10y^2+11y+70$. If one of the polynomials is $y^2-5y-14$, find the other.**

Ans:

$\text{H.C.F} = \text{y–Z}$

$\text{L.C.M} = y^3-10y^2+10y+70$

$p(y)=y^2-5y-14$

$q(y)=$

$p(y)q(y)=\text{H.C.F}\times\text{L.C.M}$

$$q(y)=\frac{\text{H.C.F}\times\text{L.C.M}}{p(y)}$$

$$q(y)=\frac{(y-7)\left(y^3-10y^2+11y+70\right)}{y^2-5y-14}$$

$$q(y)=\frac{(y-7)\left(y^3-10y^2+11y+70\right)}{y^2-7y+2y-14}$$

$$=\frac{(y-7)\left(y^3-10y^2+10y+70\right)}{y(y-7)+2(y-7)}$$

$$=\frac{y^3-10y^2+11y+70}{y+2}$$

$$\begin{array}{r|l} & y^2-12y+35 \\ \hline y+2 & y^3+10y^2+11y+70 \\ & \pm y^3\pm 2y^2 \\ \hline & -12y^2+11y+70 \\ & \mp 12y^2+24y \\ \hline & 35y+70 \\ & \pm 35y\pm 70 \end{array}$$

$q(y)=y^2-12y+35$

- **The LCM and HCF of two polynomial $p(x)$ and $q(x)$ are $36x^3(x+a)(x^3-a^3)$ and $x^2(x-a)$ respectively. If $p(x)=4x^2(x^2-a^2)$, find the other.**

Ans:

$\text{L.C.M} = 36x^3(x+a)(x^3-a^3)$

$\text{H.C.F} = x^2(x-a)$

$p(x)=4x^2(x^2-a^2)$

$q(x)$

$q(x)p(x)=\text{L.C.M}\times\text{H.C.F}$

$$q(x)=\frac{\text{L.C.M}\times\text{H.C.F}}{p(x)}$$

$$q(x)=\frac{9(x^3)(x+a)(x^3-a^3)x^2(x-a)}{4x^2(x^2-a^2)}$$

$$q(x)=\frac{9(x^3)(x+a)(x-a)(x^2+ax+a^2)(x-a)}{(x-a)(x+a)}$$

$q(x)=9x^3(x-a)(x^2+ax+a^2)$

$q(x)=9x^3(x^3-a^3)$

- **The HCF and LCM of two polynomials is $(x+a)$ and $12x^2(x+a)(x^3-a^3)$ respectively. Find the product of the two polynomials**

Ans:

$\text{H.C.F} = x+a$

$\text{L.C.M} = 12x^2(x+a)(x^2-a^2)$

$p(x)\times q(x)=?$

$p(x)\times q(x)=\text{H.C.F}\times\text{L.C.M}$

$p(x)\times q(x)=(x+a)\left[12x^2(x+a)(x^2-a^2)\right]$

$p(x)\times q(x)=12x^2(x+a)^2(x^2-a^2)$

Or

$=12x^2[x+a][x+a][x-a][x+a]$

$=12x^3(x+a)^2(x^2-a^2)$