EXERCISE 4.4

1. Find the square root of the following polynomials by factorization method.

1. $x^2-8x+16$

Ans:

$$x^2-8x+16=x^2-4x-4x+16$$
$$=x(x-4)-4(x-4)$$
$$=(x-4)(x-4)$$
$$x^2-8x-16=(x-4)^2$$

Taking square root on both sides

$$\sqrt{x^2-8x+16}=\sqrt{(x-4)^2}$$
$$=\pm\sqrt{x-4}$$

2. $9x^2+12x+4$

Ans:

$$9x^2+12x+4=9x^2+6x+6x+4$$
$$=3x(3x+2)+2(3x+2)$$
$$=(3x+2)(3x+2)$$
$$9x^2+12x+4=(3x+2)^2$$

Taking square root on both side

$$\sqrt{9x^2+12x+4}=\sqrt{(3x+2)^2}$$
$$=\pm(3x+2)$$

3. $36a^2+84a+49$

Ans:

$$36a^2+84a+49=(6a)^2+2(6a)(7)+(7)^2$$
$$36a^2+84a+49=[6a+7]^2$$

Taking square root on both side

$$\sqrt{36a^2+84a+49}=\sqrt{(6a+7)^2}$$
$$=\pm(6a+7)$$

4. $64y^2-32y+4$

Ans:

$$64y^2-32y+4=(8y)^2-2(8y)(2)+(2)^2$$
$$64y^2-32y+4=[8y-2]^2$$

Taking square root on both side

$$\sqrt{64y^2-32y+4}=\sqrt{(8y-2)^2}$$
$$64y^2-32y+4=\pm(8y-2)$$

5. $200t^2-120t+18$

Ans:

$$200t^2-120t+18=\left[100t^2-60t+9\right]$$
$$=2\left[(10t)^2-2(10t)(3)+(3)^2\right]$$
$$200t^2-120t+18=2[10t-3]^2$$
$$\sqrt{200t^2-120t+18}=\sqrt{2(10t-3)^2}$$
$$\sqrt{200t^2-120t+18}=\pm\sqrt{2}(10t-3)$$
$$=\pm(10t-3)\sqrt{2}$$

6. $40x^2+120x+90$

Ans:

$$40x^2+120x+90=10\left[4x^2+12x+9\right]$$
$$=10\left[(2x)^2+2(2x)(3)+(3)^2\right]$$
$$40x^2+120x+90=10[2x+3]^2$$

Taking square root on both side

$$\sqrt{40x^2+120x+90}=\sqrt{10(2x+3)}$$
$$=\pm\sqrt{10}(2x+3)=\pm(2x+3)\sqrt{10}$$

7. Find the square root of the following polynomials by division method.

1. $4x^4-28x^3+37x^2+42x+9$

Ans:

$$\begin{array}{r|l} & 2x^2-7x-3 \\ \hline 2x^2 & 4x^4-28x^3+37x^2+42x+9 \\ & \pm 4x^4 \\ \hline 9x^2-7x & -28x^3+37x^2+42x+9 \\ & \mp 28x^3 \pm 49x^2 \\ \hline 4x^2-14x-3 & -12x^2+42x+9 \\ & \mp 12x^2 \pm 42x \pm 9 \end{array}$$

$$\pm(2x^2-7x-3)$$

2. $121x^4-198x^3-183x^2+216x+144$

Ans:

$$\begin{array}{r|l} & 11x^2-9x+12 \\ \hline 11x^2 & 121x^4-198x^3-345x^2+216x+144 \\ & \pm 121x^4 \\ \hline 22x^2-9x & -198x^3-345x^2+216x+144 \\ & \mp 198x^3 \pm 81x^2 \\ \hline 22x^2-18x+12 & 264x^2+216x+144 \\ & \pm 264x^2 \pm 216x \pm 144 \end{array}$$

$$\pm(11x^2-9x+12)$$

3. $x^4-10x^3y+27x^2y^2-10xy^3+y^4$

Ans:

$$y^2-5xy+x^2$$

y^2	$y^4-10xy^3+27x^2y^2-10x^3y+x^4$
	$\pm y^4$
$2y^2-5xy$	$-10xy^3+27x^2y^2-10x^3y+x^4$
	$\mp 10xy^3 \pm 25x^2y^2$
$2y^2-10xy+x^2$	$2x^2y^2-10x^3y+x^4$
	$\pm 2x^2y^2 \mp 10x^3y \pm x^4$

$$\pm\left(y^2-5x+x^2\right)$$

4. $4x^4-12x^3+37x^2-42x+49$

Ans:

$$2x^2-3x+7$$

$2x^2$	$4x^4-12x^3+37x^2-42x+49$
	$\pm 4x^4$
$4x^2-3x$	$-12x^3+37x^2-42x+49$
	$\mp 12x^3 \pm 9x^2$
$4x^2-6x+7$	$28x^2-42x+49$
	$\pm 28x^2 \mp 42x \pm 49$

$$\pm\left(2x^2-3x+7\right)$$

5. **An investor's return $R(x)$ in rupees after investing x thousand rupees is given by quadratic expression:** $R(x)=-x^2+6x-8$

Ans:

Factorize the expression and find the investment levels that result in zero return

$$R(x)=-x+6x-8$$
$$=-\left[x^2-6x+8\right]=-\left[x^2-4x-2x+8\right]$$
$$=-\left[x(x-4)-2(x-4)\right]$$
$$=-1[x-4][x-2]$$
$$-1(x-4)(x-2)=0$$
$$(x-4)(x-2)=\frac{0}{-1}$$
$$(x-4)(x-2)=0$$
$$x-4=0 \qquad x-2=0$$
$$x=4 \qquad x=2$$

6. **A company's profit $P(x)$ in rupees from selling x units of a product is modeled by the cubic expression:** $P(x)=x^3-15x^2+75x-125$

Ans:

$$P(x)=x^3-15x^2+75x-125$$
$$(a-b)^3=a^3-b^3-3ab(a-b)$$
$$P(x)=(x)^3-(5)^3-3\left(x^2\right)(5)+3(5)(x)^2$$
$$P(x)=(x-5)^3$$
$$P(x)=0$$
$$0=(x-5)^3$$

Taking cube root on both side

$$\sqrt[3]{(x-5)^3}=\sqrt[3]{0}$$
$$x-5=0$$
$$x=5$$

7. **The potential energy $V(x)$ in an electric field varies as a cubic function of distance x, given by:** $V(x)=2x^3-6x^2+4x$

Ans:

Determine where the potential energy is zero

$$V(x)=2x^3-6x^2+4x$$
$$V(x)=x\left[2x^2-6x+4\right]$$
$$=x\left[2x^2-4x-2x+4\right]$$
$$=x\left[2x(x-2)-2(x-2)\right]$$
$$=x[x-2][2x-2]$$
$$V(x)=0$$
$$0=x(x-2)(2x-2)$$
$$x=0,\ x-2=0,\ \ 2x-2=0$$
$$x=2, \quad 2x=2$$
$$x=\frac{2}{2}$$
$$x=1$$

$0, 2$ and 1

8. **In structural engineering, the deflection $Y(x)$ of a beam is given**

by: $Y(x) = 2x^2 - 8x + 6$

This equation gives the vertical deflection at any point *x* along the beam. Find the points of zero deflection.

Ans:

$Y(x) = 2x^2 - 8x + 6$

$= 2x^2 - 2x - 6x + 6$

$= 2x(x-1) - 6(x-1)$

$= (x-1)(2x-6)$

$Y(x) = 0$

$0 = (x-1)(2x-6)$

$x = 1 = 0, \qquad 2x - 6 = 0$

$x = 1 \qquad 2x = 6$

$x = \frac{6}{2}$

$x = 3$