EXERCISE 5.2

Q.1 Maximize $f(x,y)=2x+5y$; subject to the constraints

$2y-x\le 8;\ x-y\le 4;\ x\ge 0;\ y\ge 0$

Ans:

$2y-x\le 8$

$-x+2y\le 8$

Associated

$2y-x=8$

Dividing by 8

$\frac{2y}{8}-\frac{x}{8}=\frac{8}{8}$

$x-y\le 4$

Associated

$x-y=4$

Dividing by 4

$\frac{x}{4}-\frac{y}{4}=\frac{4}{4}$

$\frac{x}{4}-\frac{y}{4}=1$

$\frac{-x}{8}+\frac{y}{4}=1$

x int $(-8,0)$

y int $(0,4)$

Test point

$(x,y)=(0,0)$

$2(0)-0\le 8$

$0\le 8$

True toward the origin

x int $(4,0)$

y int $(0,-4)$

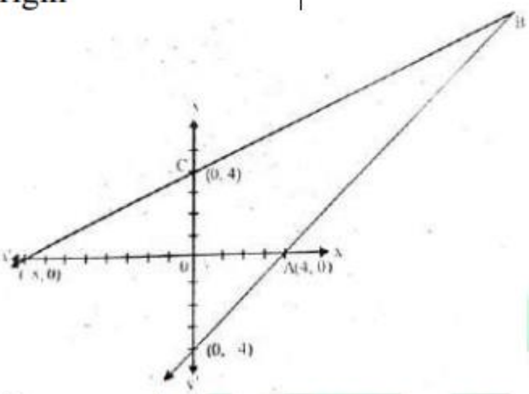
Test point

$(x,y)=(0,0)$

$0-0\le 4$

$0\le 4$

True toward region



Corner points

Adding equation (i) and (ii)

$-x+2y=8$

$x-y=4$

$y=12$

Put in equation (ii)

$x-12=4$

$x=4+12$

$x=16$

$(0,0)(4,0)(0,4)(16,12)$

For minimize

$f(x,y)=2x+5y$

$f(0,0)=2(0)+5(0)=0$

$f(4,0)=2(4)+5(0)=8$

$f(0,4)=2(0)+5(4)$

$f(0,4)=20$

$f(16,12)=2(16)+5(12)$

$=32+60=92$

Hence maximize is at $(16,12)$

Q.2 Maximize $f(x,y)=x+3y$; subject to the constraints

$2x+5y\le 30;\ 5x+4y\le 20;\ x\ge 0;\ y\ge 0$

Ans:

Associated

$2x+5y=30$

Dividing by 30

$\frac{2x}{30}+\frac{5y}{30}=\frac{30}{30}$

$\frac{x}{15}+\frac{y}{6}=1$

x int $(15,0)$

y int $(0,6)$

Test point

$(x,y)=(0,0)$

$2(0)+5(0)\le 30$

$0\le 30$

True toward the origin

$5x+4y=20$

Dividing by 20

$\frac{5x}{20}+\frac{4y}{20}=\frac{20}{20}$

$\frac{x}{4}+\frac{y}{5}=1$

x int $(4,0)$

y int $(0,5)$

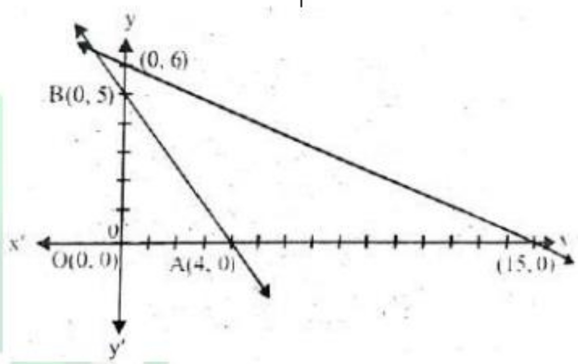
Test point

$(x,y)=(0,0)$

$5(0)+4(0)\le 20$

$0\le 20$

True toward region



Corner points

$(0,0),(4,0),(0,5)$

Maximize

$f(x,y)=x+3y$

$f(0,6)=0+3(0)$

$f(0,6)=0$

$f(4,0)=4+3(0)=4$

$f(0,5)=0+3(5)=15$

Hence maximize at $(0,5)$

Q.3 Maximize $f(x,y)=x+3y$; subject to the constraints

$2x+y\le 4;\ 4x-y\le 4;\ x\ge 0;\ y\ge 0$

Ans:

Associated

$2x+y=4$

Dividing by 4

$\frac{2x}{4}+\frac{y}{4}=\frac{4}{4}$

$\frac{x}{2}+\frac{y}{4}=1$

x int $(2,0)$

y int $(0,4)$

$4x-y=4$

Dividing by 4

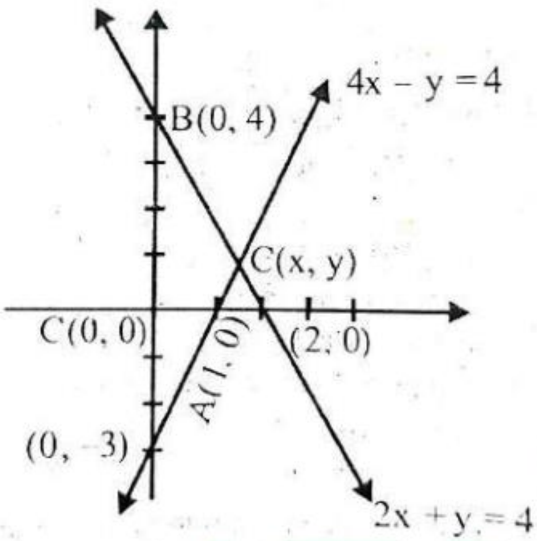
$\frac{4x}{4}-\frac{y}{4}=\frac{4}{4}$

$\frac{x}{1}-\frac{y}{4}=1$

x int $(1,0)$

y int $(0,4)$

Test point $(x,y)=(0,0)$ $2(0)+0\leq 4$ $0\leq 4$ True toward the origin	Test point $(x,y)=(0,0)$ $4(0)-0\leq 4$ $0\leq 4$ True toward region



Corner points

Adding equation (i) and (ii)

$$\begin{array}{r} 2x+y=4 \\ 4x-y=4 \\ \hline 6x=8 \end{array}$$

$x=\frac{8}{6}$

$x=\frac{4}{3}$

Put in (i)

$2\left(\frac{4}{3}\right)+y=4$

$\frac{8}{3}+y=4$

$y=4-\frac{8}{3}$

$y=\frac{12-8}{3}$

$y=\frac{4}{3}$

Here are corner points

$(0,0)(1,0)\left(\frac{4}{3},\frac{4}{-}3\right);(0,4)$

Form maximize

$z=2x+3y$

$z=2(0)+3(0)$

$z=0$

$z=2$

$z=2\left[\frac{4}{3}\right]+3\left[\frac{4}{3}\right]$

$z=\frac{8}{3}+\frac{12}{3}$

$z=\frac{8+12}{3}$

$z=\frac{20}{3}=6.67$

$z=2(0)+3(4)$

$z=12$

Hence maximize is at $(0,4)$

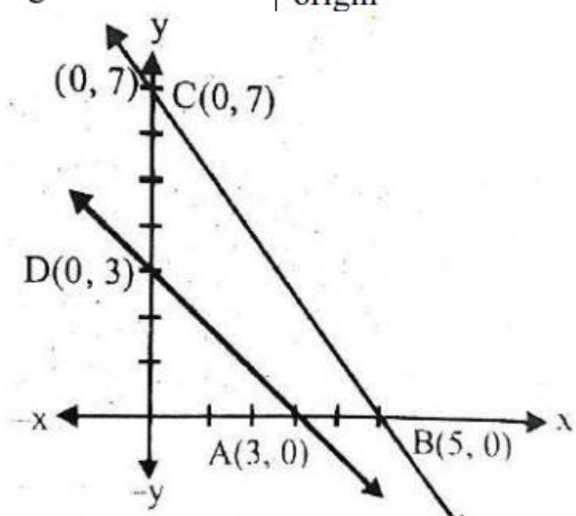
Q.4 Minimize $z=2x+y$; **subject to the constraints**

$x+y\geq 3;\ 7x+5y\leq 35;\ x\geq 0;\ y\geq 0$

Ans:

Associated

$x+y=3$ Dividing by 3 $\frac{x}{3}+\frac{y}{3}=\frac{3}{3}$ $\frac{x}{3}+\frac{y}{3}=1$ x int $(3,0)$ y int $(0,3)$ Test point $(x,y)=(0,0)$ $0+0\geq 3$ $0\leq 3$ False away from origin	$7x+5=35$ Dividing by 35 $\frac{7x}{35}+\frac{5y}{35}=\frac{35}{35}$ $\frac{x}{5}+\frac{y}{7}=1$ x int $(5,0)$ y int $(0,7)$ Test point $(x,y)=(0,0)$ $7(0)+5(0)\leq 35$ $0\leq 35$ It is true towards origin



Corner points

Hence corner points

$(3,0)(0,3)(5,0)(0,7)$

Minimize

$z = 2x + y$

$z = 2(3) + 0 = 6$

$z = 2(0) + 3 = 3$

$z = 2(5) + 0 = 10$

$z = 2(0) + 7 = 7$

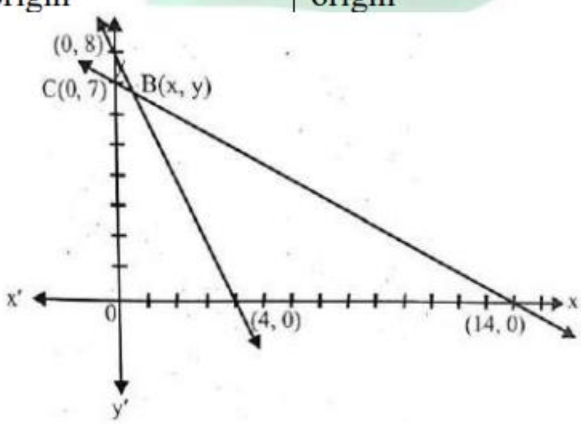
Hence minimize $(0,3)$

Q.5 Maximize the function defined as; $f(x,y) = 2x + 3y$ subject to the constraints:
$2x + y \le 8;\ x + 2y \le 14;\ x \ge 0;\ y \ge 0$

Ans:

Associated

$2x + y = 8$ ______(i)	$x + 2y = 14$ ______(ii)
Dividing by 8	Dividing by 14
$\frac{2x}{8} + \frac{y}{8} = \frac{8}{8}$	$\frac{x}{14} + \frac{2y}{14} = \frac{14}{14}$
$\frac{x}{4} + \frac{y}{8} = 1$	$\frac{x}{14} + \frac{y}{7} = 1$
x int $(4,0)$	x int $(14,0)$
y int $(0,8)$	y int $(0,7)$
Test point	Test point
$(x,y) = (0,0)$	$(x,y) = (0,0)$
$2(0) + 0 \le 8$	$0 + 2(0) \le 14$
$0 \le 8$	$0 \le 14$
It is true towards origin	It is true towards origin



Corner points

(ii) by the subtraction by (i)

$$
\begin{array}{r}
2x + 4y = 25 \\
\underline{\pm 2x \pm y = \pm 8} \\
3y = 20
\end{array}
$$

$y = \frac{20}{3}$

Put it in (ii)

$x + 2\left[\frac{20}{3}\right] = 14$

$x + \frac{40}{3} = 14$

$x = 14 - \frac{40}{3} = \frac{42-40}{3}$

$x = \frac{2}{3}$

Hence corner points are

$(0,0)(4,0)(0,7)\left(\frac{2}{3}, \frac{20}{3}\right)$

For maximize

$f(x,y) = 2x + 3y$

$f(0,0) = 2(0) + 3(0) = 0$

$f(4,0) = 2(4) + 3(0) = 8$

$f(0,7) = 2(0) + 3(7) = 21$

$f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right) = \frac{4}{3} + \frac{60}{3}$

$= \frac{4+60}{3} = \frac{64}{3} = 21.33$

Hence maximize at $\left(\frac{2}{3}, \frac{20}{3}\right)$

Q.6 Find minimum and maximum values of $z = 3x + y;$ subject to the constraints:
$3x + 5y \ge 15;\ x + 6y \ge 9;\ x \ge 0;\ y \ge 0$

Ans:

Associated

$3x + 5y = 15$ ______(i)	$x + 6y = 6$ ______(ii)
Dividing by 15	Dividing by 9
$\frac{3x}{15} + \frac{5y}{15} = \frac{15}{15}$	$\frac{x}{9} + \frac{6y}{9} = \frac{9}{9}$
$\frac{x}{5} + \frac{y}{3} = 1$	$\frac{x}{9} + \frac{2y}{3} = 1$
x int $(5,0)$	x int $(9,0)$
y int $(0,3)$	y int $(0,2)$
Test point	Test point

$(x,y)=(0,0)$

$3(0)+5(0)\geq 15$

$0\leq 15$

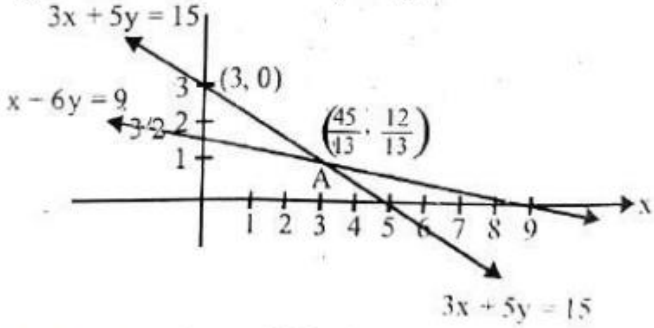
False it is away from origin

$(x,y)=(0,0)$

$0+6(0)\geq 9$

$0\leq 9$

False it is away from origin



Corner points

x by (ii) by (iii) subtraction with,

$$3x+18y=24$$

$$\underline{\pm 3x\pm 5y=\pm 15}$$

$$13y=12$$

$$y=\frac{12}{13}$$

Put it in (ii)

$$x+6\left(\frac{12}{13}\right)=9$$

$$x-\frac{72}{13}=9$$

$$x=9-\frac{72}{13}=\frac{117-72}{13}$$

$$x=\frac{45}{13}$$

Hence corner points are

$$z=3x+y$$

$$z=3(9)+0=27$$

$$z=3\left[\frac{45}{3}\right]+\frac{12}{13}=11.3$$

$$z=3(0)+3=3$$

Minimize $(0,3)$

Maximize $(9,0)$