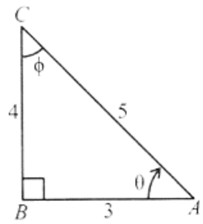
EXERCISE 6.2

For each of the following right-angled triangles, find the trigonometric ratios:



(a)

(i) $\sin\theta$

Ans:

$$\sin\theta = \frac{4}{5}$$

(ii) $\cos\theta$

Ans:

$$\cos\theta = \frac{3}{5}$$

(iii) $\tan\theta$

Ans:

$$\tan\theta = \frac{4}{3}$$

(iv) $\sec\theta$

Ans:

$$\sec\theta = \frac{5}{3}$$

(v) $\cos ec\theta$

Ans:

$$\text{cosec}\,\theta = \frac{5}{4}$$

(vi) $\cot\phi$

Ans:

$$\cot\phi = \frac{4}{3}$$

(vii) $\tan\phi$

Ans:

$$\tan\phi = \frac{3}{4}$$

(viii) $\cos ec\phi$

Ans:

$$\text{cosec}\,\phi = \frac{5}{3}$$

(ix) $\sec\phi$

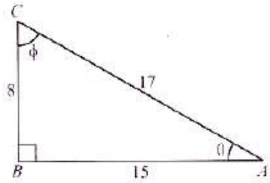
Ans:

$$\sec\phi = \frac{5}{4}$$

(x) $\cos\phi$

Ans:

$$\cos\phi = \frac{4}{5}$$



(b)

(i) $\sin\theta$

Ans:

$$\sin\theta = \frac{8}{17}$$

(ii) $\cos\theta$

Ans:

$$\cos\theta = \frac{15}{17}$$

(iii) $\tan\theta$

Ans:

$$\tan\theta = \frac{8}{15}$$

(iv) $\sec\theta$

Ans:

$$\sec\theta = \frac{17}{15}$$

(v) $\cos ec\theta$

Ans:

$$\text{cosec}\,\theta = \frac{17}{8}$$

(vi) $\cot\phi$

Ans:

$$\cot\phi = \frac{8}{15}$$

(vii) $\tan\phi$

Ans:

$$\tan\phi = \frac{15}{8}$$

(viii) $\cos ec\phi$

Ans:

$$\text{cosec}\,\phi = \frac{17}{15}$$

(ix) $\sec\phi$

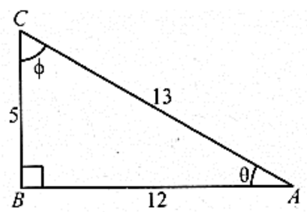
Ans:

$$\sec\phi = \frac{17}{8}$$

(x) $\cos\phi$

Ans:

$$\cos\phi = \frac{8}{17}$$

(c)


(i) $\sin\theta$

Ans:

$$\sin\theta = \frac{5}{13}$$

(ii) $\cos\theta$

Ans:

$$\cos\theta = \frac{12}{13}$$

(iii) $\tan\theta$

Ans:

$$\tan\theta = \frac{5}{12}$$

(iv) $\sec\theta$

Ans:

$$\sec\theta = \frac{13}{12}$$

(v) $\cos ec\theta$

Ans:

$$\text{cosec}\,\theta = \frac{13}{15}$$

(vi) $\cot\phi$

Ans:

$$\cot\phi = \frac{5}{12}$$

(vii) $\tan\phi$

Ans:

$$\tan\phi = \frac{12}{5}$$

(viii) $\cos ec\phi$

Ans:

$$\text{cosec}\,\phi = \frac{13}{12}$$

(ix) $\sec\phi$

Ans:

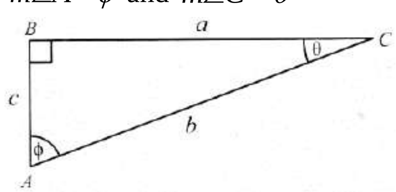
$$\sec\phi = \frac{13}{5}$$

(x) $\cos\phi$

Ans:

$$\cos\phi = \frac{5}{13}$$

Q.1 For the following right-angled triangle ABC **find the trigonometric ratios for which** $m\angle A = \phi$ and $m\angle C = \theta$



(i) $\sin\theta$

Ans:

$$\sin\theta = \frac{c}{b}$$

(ii) $\cos\theta$

Ans:

$$\cos\theta = \frac{a}{b}$$

(iii) $\tan\theta$

Ans:

$\tan\theta = \frac{c}{a}$

(iv) $\sin\phi$

Ans:

$\sin\phi = \frac{a}{b}$

(v) $\cos\phi$

Ans:

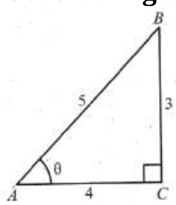
$\cos\phi = \frac{c}{b}$

(vi) $\tan\phi$

Ans:

$\tan\phi = \frac{a}{c}$

Q.2 Considering the adjoining triangle *ABC*, verify that:



(i) $\sin\theta \operatorname{cosec}\theta = 1$

Ans:

L.H.S $\sin\theta \operatorname{cosec}\theta$

$\sin\theta = \frac{3}{5}$ $\quad$ $\operatorname{cosec}\frac{5}{3}$

$= \sin\theta \quad \operatorname{cosec}\theta$

$\frac{3}{5}\times\frac{5}{3}$

1 Hence proved

(ii) $\cos\theta \sec\theta = 1$

Ans:

$\cos\theta = \frac{4}{5}$ $\quad \sec\theta = \frac{5}{4}$

$\frac{4}{5}\times\frac{5}{4} = 1$

$1 = 1$

(iii) $\tan\theta \cot\theta = 1$

Ans:

$\frac{3}{4}\times\frac{4}{3} = 1 \Rightarrow 1 = 1$

Q.3 Fill in the blanks.

(i) $\sin 30° = \sin(90° - 60°) =$ ___________

Ans: $\sin 30° = \sin(90° - 60°) = \cos 60°$

(ii) $\cos 30° = \cos(90° - 60°) =$ ___________

Ans: $\cos 30° = \cos(90° - 60°) = \sin 60°$

(iii) $\tan 30^\circ = \tan(90^\circ - 60^\circ) =$ ___________

Ans: $\tan 30^\circ = \tan(90^\circ - 60^\circ) = \cot 30^\circ$

(iv) $\tan 60^\circ = \tan(90^\circ - 30^\circ) =$ ___________

Ans: $\tan 60^\circ = \tan(90^\circ - 30^\circ) = \cos 30^\circ$

(v) $\sin 60^\circ = \sin(90^\circ - 30^\circ) =$ ___________

Ans: $\sin 60^\circ = \sin(90^\circ - 30^\circ) = \sin 30^\circ$

(vi) $\cos 60^\circ = \cos(90^\circ - 30^\circ) =$ ___________

Ans: $\cos 60^\circ = \cos(90^\circ - 30^\circ) = \sin 30^\circ$

(vii) $\sin 45^\circ = \sin(90^\circ - 45^\circ) =$ ___________

Ans: $\sin 45^\circ = \sin(90^\circ - 45^\circ) = \cos 45^\circ$

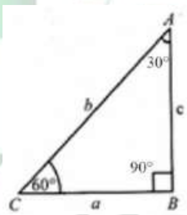
(viii) $\tan 45^\circ = \sin(90^\circ - 45^\circ) =$ ___________

Ans: $\tan 45^\circ = \sin(90^\circ - 45^\circ) = \cot 45^\circ$

(ix) $\cos 45^\circ = \cos(90^\circ - 45^\circ) =$ ___________

Ans: $\cos 45^\circ = \cos(90^\circ - 45^\circ) = \sin 45^\circ$

Q.4 In a right angled triangle ABC**,** $m\angle B = 90^\circ$ **and** C **is an acute angle of 60°. Also** $\sin m\angle = \frac{a}{b}$, **then find the following trigonometric ratios:**



(i) $\frac{m\overline{BC}}{m\overline{AB}}$

Ans: $\frac{m\overline{BC}}{m\overline{AB}} = \frac{a}{c}$

(ii) $\cos 60^\circ$

Ans: $\cos 60^\circ = \frac{a}{b}$

(iii) $\tan 60^\circ$

Ans: $\tan 60^\circ = \frac{c}{a}$

(iv) $\operatorname{cosec}\frac{\pi}{3}$

Ans: $\operatorname{cosec}\frac{\pi}{3} = \frac{b}{c}$

(v) $\cot 60^\circ$

Ans: $\cot 60^\circ = \frac{a}{c}$

(vi) $\sin 30^\circ$

Ans: $\sin 30^\circ = \frac{a}{b}$

(vii) $\cos 30^\circ$

Ans: $\cos 30^\circ = \frac{c}{b}$

(viii) $\tan\frac{\pi}{6}$

Ans: $\tan\frac{\pi}{6}=\frac{a}{c}$

(ix) $\sec 30^\circ$

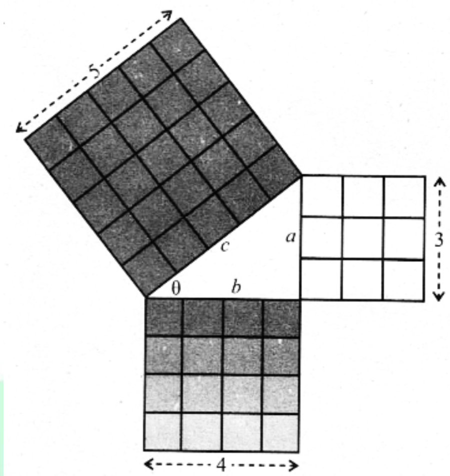
Ans: $\sec 30^\circ=\frac{b}{c}$

(x) $\cot 30^\circ$

Ans: $\cot 30^\circ=\frac{c}{a}$

Trigonometric Identities

Fundamental Trigonometric Identities



We shall consider some of the fundamental identities used in trigonometry. The key to these basic identities is the Pythagoras theorem in geometry.

"The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides".

$$c^2=a^2+b^2$$
$$5^2=3^2+4^2$$
$$25=9+16$$

In the given figure:

The perpendicular equals to the length $'a'$ base equals to the length $'b'$ and hypotenuse equals to the length $'c'$.

By Pythagoras Theorem, we have

$\boxed{a^2+b^2=c^2}$ …(i)

$\frac{a^2}{c^2}+\frac{b^2}{c^2}=\frac{c^2}{2}$ (Dividing by c^2)

$\boxed{\sin^2\theta+\cos^2\theta=1}$ …(ii)

$a^2+b^2=c^2$

$\frac{a^2}{b^2}+\frac{b^2}{b^2}=\frac{c^2}{b^2}$ Dividing equation (i) by b^2, we have

$\boxed{\tan^2\theta+1=\sec^2\theta}$ …(iii)

$\frac{a^2}{a^2}+\frac{b^2}{a^2}=\frac{c^2}{a^2}$ Dividing equation (i) by a^2, we have

$\boxed{1+\cot^2\theta=\text{cosec}^2\theta}$ …(iv)

The identities (ii), (iii) and (iv) are known as Pythagoras identities.

Example 10:

Show that $\left(\sec^2\theta-1\right)\cos^2\theta=\sin^2\theta$

Solution:

$\text{L.H.S}=\left(\sec^2\theta-1\right)\cos^2\theta$