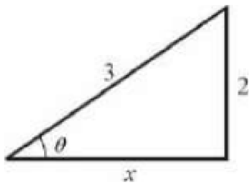
EXERCISE 6.3

Q.1 **If θ lies in first quadrant, find the remaining trigonometric ratios of θ**

(i) $\sin\theta = \dfrac{2}{3}$

Ans:

$$(\text{Hyp})^2 = (\text{Prep})^2 + (\text{base})^2$$



$$(3)^2 = 2^2 + x^2$$

$$9 = 4 + x^2$$

$$9 - 4 = x^2$$

$$\sqrt{x^2} = \sqrt{5}$$

$$x = \sqrt{5}$$

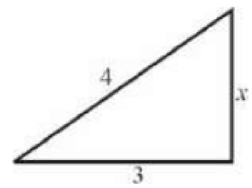
$\cos\theta = \dfrac{\sqrt{5}}{3}$ $\qquad$ $\sec\theta = \dfrac{3}{\sqrt{5}}$

$\sin\theta = \dfrac{2}{3}$ $\qquad$ $\operatorname{cosec}\theta = \dfrac{3}{2}$

$\tan\theta = \dfrac{2}{\sqrt{5}}$ $\qquad$ $\cot\theta = \dfrac{\sqrt{5}}{2}$

(ii) $\cos\theta = \dfrac{3}{4}$

Ans:



$$(4)^2 = (3)^2 + x^2$$

$$16 = 9 + x^2$$

$$16 - 9 = x^2$$

$$\sqrt{x^2} = \sqrt{7}$$

$x=\sqrt{7}$

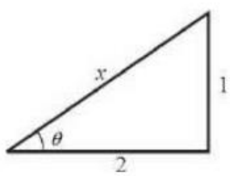
$\sin\theta=\frac{\sqrt{7}}{4}$ $\quad$ $\operatorname{cosec}\theta=\frac{4}{\sqrt{7}}$

$\cos\theta=\frac{3}{4}$ $\quad$ $\sec\theta=\frac{4}{3}$

$\tan\theta=\frac{\sqrt{7}}{3}$ $\quad$ $\cot\theta=\frac{3}{\sqrt{7}}$

(iii) $\tan\theta=\frac{1}{2}$

Ans:



$x^2=(2)^2+(1)^2$

$x^2=4+1$

$\sqrt{x^2}=\sqrt{5}$

$x=\sqrt{5}$

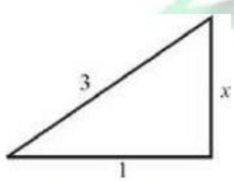
$\sin\theta=\frac{1}{5}$ $\quad$ $\operatorname{cosec}\theta=\frac{\sqrt{5}}{1}$

$\cos\theta=\frac{2}{\sqrt{5}}$ $\quad$ $\sec\theta=\frac{\sqrt{5}}{2}$

$\tan\theta=\frac{1}{2}$ $\quad$ $\cot\theta=\frac{2}{1}=2$

(iv) $\sec\theta=3$

Ans:



$(3)^2=(1)^2+x^2$

$9=1+x^2$

$9-1=x^2$

$8=x^2$

$\sqrt{x^2}=\sqrt{8}$

$x=2\sqrt{2}$

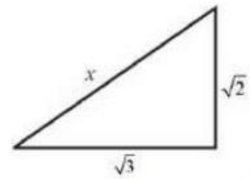
$\sin\theta=\frac{2\sqrt{2}}{3}$ $\quad$ $\operatorname{cosec}\theta=\frac{3}{2\sqrt{2}}$

$\cos\theta=\frac{1}{3}$ $\quad$ $\sec\theta=3$

$\tan\theta=\frac{2\sqrt{3}}{1}=2\sqrt{3}$ $\quad$ $\cot\theta=\frac{1}{2\sqrt{3}}$

(v) $\cot\theta=\sqrt{\frac{3}{2}}$

Ans:



$x^2=\left(\sqrt{2}\right)^2+\left(\sqrt{3}\right)^2$

$x^2=2+3$

$\sqrt{x^2}=\sqrt{5}$

$x=\sqrt{5}$

$\sin\theta=\frac{\sqrt{2}}{\sqrt{5}}$ $\quad$ $\operatorname{cosec}\theta=\frac{\sqrt{5}}{\sqrt{2}}$

$\cos\theta=\frac{\sqrt{3}}{\sqrt{5}}$ $\quad$ $\sec\theta=\frac{\sqrt{5}}{\sqrt{3}}$

$\tan\theta=\frac{\sqrt{2}}{\sqrt{3}}$ $\quad$ $\cot\theta=\frac{\sqrt{3}}{\sqrt{2}}$

Prove the following trigonometric identities:

Q.2 $(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta$

Ans:

L.H.S $(\sin\theta+\cos\theta)^2$

$(\sin\theta+\cos\theta)^2=\cos^2\theta+\sin^2\theta+2\cos\theta sub\theta$

As we know $\cos^2\theta+\sin^2\theta=1$

$1+2\cos\theta\sin\theta$

L.H.S = R.H.S

Q.3 $\frac{\cos\theta}{\sin\theta}=\frac{1}{\tan\theta}$

Ans:

R.H.S $\frac{1}{\tan\theta}$

$\frac{1}{\tan\theta}=1\div\tan\theta$

$=1\div\frac{\sin\theta}{\cos\theta}$

$=1\times\frac{\cos\theta}{\sin\theta}=\frac{\cos\theta}{\sin\theta}$

L.H.S = R.H.S

Q.4 $\dfrac{\sin\theta}{\operatorname{cosec}\theta}+\dfrac{\cos\theta}{\sec\theta}=1$

Ans:

$$\text{L.H.S} = \frac{\sin\theta}{\operatorname{cosec}\theta}+\frac{\cos\theta}{\sec\theta}$$
$$=\sin\theta\div\operatorname{cosec}\theta+\cos\theta\div\sec\theta$$
$$=\sin\theta\times\sin\theta+\cos\theta\times\sec\theta$$
$$=\sin^2\theta+\cos^2\theta$$
$$=1$$
$$\text{L.H.S}=\text{R.H.S}$$

Q.5 $\cos^2\theta-\sin^2\theta=2\cos^2\theta-1$

Ans:

$$\text{L.H.S } \cos^2\theta-\sin^2\theta$$
$$=\cos^2\theta-\left[1-\cos^2\theta\right]$$
$$=\cos^2\theta-1+\cos^2\theta$$
$$=2\cos^2\theta-1$$
$$\text{L.H.S}=\text{R.H.S}$$

Q.6 $\cos^2\theta-\sin^2\theta=1-2\sin^2\theta$

Ans:

$$\text{L.H.S } \cos^2\theta-\sin^2\theta$$
$$\cos^2\theta=1-\sin^2\theta$$
$$1-\sin^2\theta-\sin^2\theta$$
$$1-2\sin^2\theta$$

Q.7 $\dfrac{1-\sin\theta}{\cos\theta}=\dfrac{\cos\theta}{1+\sin\theta}$

Ans:

$$\text{L.H.S } \frac{1-\sin\theta}{\cos\theta}$$

Multiplying and dividing by $1+\sin\theta$

$$=\frac{1-\sin\theta}{\cos\theta}\times\frac{1+\sin\theta}{1+\sin\theta}$$
$$=\frac{1-\sin\theta}{\cos\theta(1+\sin\theta)} \quad 1-\sin^2\theta=\cos^2\theta$$
$$=\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{\cos\theta\times\cos\theta}{\cos\theta(1+\sin\theta)}$$
$$=\frac{\cos\theta}{1+\sin\theta}$$
$$\text{L.H.S}=\text{R.H.S}$$

Q.8 $(\sec\theta-\tan\theta)^2=\dfrac{1-\sin\theta}{1+\sin\theta}$

Ans:

$$\text{L.H.S}$$
$$(\sec\theta-\tan\theta)^2$$
$$=\left[\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right]^2=\left[\frac{1-\sin\theta}{\cos\theta}\right]^2$$
$$=\frac{(1-\sin\theta)}{\cos^2\theta}=\frac{(1-\sin\theta)(1-\sin\theta)}{1-\sin^2 gq}$$
$$=\frac{(1-\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}=\frac{1-\sin\theta}{1+\sin\theta}$$

Q.9 $(\tan\theta+\cot\theta)^2=\sec^2\theta\operatorname{cosec}^2\theta$

Ans:

$$\text{L.H.S}$$
$$=(\tan\theta+\cot\theta)^2$$
$$=\left[\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right]^2=\left[\frac{\sin^2\theta+\cos\theta}{\cos\theta\sin\theta}\right]^2$$
$$=\left[\frac{1}{\cos\theta\sin\theta}\right]^2=\left[\frac{1}{\cos\theta}\times\frac{1}{\sin\theta}\right]$$
$$=\frac{1}{\cos^2\theta}\times\frac{1}{\sin^2\theta}$$
$$=\sec^2\theta\operatorname{cosec}^2\theta$$

Hence proved

Q.10 $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\tan\theta+\sec\theta$

Ans:

$$1+\tan^2\theta=\sec^2\theta$$
$$1=\sec^2\theta-\tan^2\theta$$
$$1=(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)$$
$$=\frac{(\tan\theta+\sec\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{\tan\theta-\sec\theta+1}$$
$$=\frac{(\sec\theta+\tan\theta)(1-\sec\theta-\tan\theta)}{\tan\theta-\sec\theta+1}$$
$$=\frac{(\sec\theta+\tan\theta)(1-\sec\theta-\tan\theta)}{1-\sec\theta+\tan\theta}$$
$$=\sec\theta+\tan\theta$$

Or

$$=\tan\theta+\sec\theta$$

Q.11 $\sin^3\theta-\cos^3\theta=(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$

Ans:

$$\text{L.H.S}$$
$$\sin^3\theta-\cos^3\theta$$
$$(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)a^3-b^3=(a-b)(a^2+ab+b^2)$$
$$(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)$$
$$(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$$

Hence proved

Q.12 $\sin^6\theta - \cos^6\theta = \left(\sin^2\theta - \cos^2\theta\right)\left(1 - \sin^2\theta\cos^2\theta\right)$

Ans:

L.H.S

$\sin^6\theta - \cos^6\theta$

$\left(\sin^2\theta\right)^3 - \left(\cos^2\theta\right)^3$

$\left(\sin^2\theta - \cos^2\theta\right)\left(\sin^2\theta\right)^2 + \left(\cos^2\theta\right)^2 + 2\sin^2\cos^2\theta - 2\sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta$

$\left(\sin^2\theta - \cos^2\theta\right)\left[\left(\sin^2\theta + \cos^2\theta\right)^2 - \sin^2\theta\cos^2\theta\right]$

$\left[\sin^2\theta - \cos^2\theta\right]\left[(1)^2 - \sin^2\theta\cos^2\theta\right]$

$\left[\sin^2\theta - \cos^2\theta\right]\left[1 - \sin^2\theta\cos^2\theta\right]$