EXERCISE 7.2

Q.1 **Find the slope and inclination of the line joining the points:**

(i) $(-2,4);(5,11)$

Ans:

$x_1=-2,\ y_1=4,\ x_2=5,\ y_2=11$

$M=\frac{y_2-y_1}{x_2-x_1}$

$M=\frac{11-4}{5+2}=\frac{7}{7}$

$M=1$

$\tan\theta=m$

$\theta=\tan^{-1}m$

$\theta=\tan^{-1}1$

$\theta=45°$

(ii) $(3,-2);(2,7)$

Ans:

$x_1=3,\ y_1=-2,\ x_2=2,\ y_2=7$

$m=\frac{y_2-y_1}{x_2-x_1}$

$m=\frac{7-(-2)}{2-3}=\frac{7+2}{-1}=\frac{9}{-1}$

$m=-9$

$\tan\alpha=m$

$\alpha=\tan^{-1}(-9)$

$\alpha=180°-83.6°$

$\alpha=96.34°$

(iii) $(4,6);(4,8)$

Ans:

$x_1=4,\ y_1=6,\ x_2=6,\ y_2=8$

$m=\frac{y_2-y_1}{x_2-y_1}$

$m=\frac{8-6}{4-4}$

$m=\alpha$ undefined

$\tan\alpha=m$

$\alpha=\tan^{-1}(\alpha)$

$\alpha=90°$

Q.2 **By means of slopes, show that the following points lie on the same line:**

(i) $A(-1,-3);B(1,5);C(2,9)$

Ans:

$A(-1,-3),B(1,5)$

$x_1=-1,\ y_1=-3,\ x_2=1,\ y_2=5$

Slope of $AB=\ m=\frac{y_2-y_1}{x_2-y_1}$

$m_1=\frac{5-(-3)}{1-(-1)}=\frac{8}{2}$

$m_1=4$

$B(1,5),C(2,9)$

$x_1=1,\ y_1=5,\ x_2=2,\ y_2=9$

Slope $BC=m_2=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{9-5}{2-1}=\frac{4}{1}=4$

$A(-1,-3),C(2,9)$

Slope of $AC=m_3=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{9-(-3)}{2-(-1)}=\frac{12}{3}=4$

$m_1=m_2=m_3$

Hence they m_1,m_3 and m_2 are equal to each other so points lies on the same line slope

(ii) $P(4,-5);Q(7,5);R(10,15)$

Ans:

$P(4,-5),Q(7,5).$

$x_1=4,\ y_1=-4,\ x_2=7,\ y_2=5$

Slope of $PQ=m_1=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{5-(-5)}{7-4}=\frac{9}{3}$

$\frac{5+5}{3}=\frac{10}{3}$

$Q(7,5), R(10,15)$

$x_1 = 7,\ y_1 = 5,\ x_2 = 10,\ y_2 = 15$

Slope of $QR = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{15-5}{10-3} = \dfrac{10}{3}$

Slope of $PR = m_3 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$P(4,-5), R(10,15)$

$x_1 = 4,\ y_1 = -4,\ x_2 = 10,\ y_2 = 15$

$m_3 = \dfrac{15-(-5)}{10-4} = \dfrac{15+5}{6} = \dfrac{20}{6} = \dfrac{10}{3}$

$m_1 = m_2 = m_3$

Hence the points lie on same line

(iii) $L(-4,6); M(3,8); N(10,10)$

Ans:

$L(-4,6), M(3,8)$

$x_1 = -4,\ y_1 = 6,\ x_2 = 3,\ y_2 = 8$

Slope of $LM = m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{8-6}{3-(-4)} = \dfrac{2}{7}$

$M(3,8), N(10,10)$

$x_1 = 3,\ y_1 = 8,\ x_2 = 10,\ y_2 = 10$

Slope of $MN = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{10-8}{10-3} = \dfrac{2}{7}$

$m_2 = \dfrac{2}{7}$

$L(-4,6), N(10,10)$

$x_1 = -4,\ y_1 = 6,\ x_2 = 10,\ y_2 = 10$

Slope of $LN = m_3 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{10-6}{10-(-4)}$

$m_3 = \dfrac{2}{7}$

$m_1 = m_2 = m_3$

So the points lies on same line

(iv) $X(a,2b); Y(c,a+b); Z(2c-a,2a)$

Ans:

$X(a,2b), Y(c,a+b)$

$x_1 = a,\ y_1 = 2b,\ x_2 = c,\ y_2 = a+b$

Slope of $XY = m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \dfrac{a+b-2b}{c-a} = \dfrac{a-b}{c-a}$

$X(a,2b), z(2c-a,2a)$

$x_1 = a,\ y_1 = 2b,\ x_2 = 2c-a,\ y_2 = 2a$

Slope of $XZ = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{2a-2b}{2c-a-2b} = \dfrac{2a-2b}{2c-a-a}$

$m_2 = \dfrac{2(a-b)}{2c-2a}$

$m_2 = \dfrac{a-b}{c-a}$

$x_1 = c,\ y_1 = a+b,\ x_2 = 2c-a,\ y_2 = 2a$

Slope of $YZ = m_3 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_3 = \dfrac{2a-(a+b)}{2(-a-(c))}$

$m_3 = \dfrac{2a-a-b}{2c-a-c}$

$m_3 = \dfrac{a-b}{c-a}$

$m_1 = m_2 = m_3$

So, the points lies on same line

Q.3 **Find k so that the line joining $A(7,3); B(k,-6)$ and the line joining $C(-4,5); D(-6,4)$ are:**

(i)

arallel

(ii)

erpendicular

Ans:

$A(7,3), B(k-6)$

$x_1 = 7,\ y_1 = 3,\ x_2 = k,\ y_2 = -6$

Slope of $AB = m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-6-3}{k-7} = \dfrac{-9}{k-7}$

$C(-4,5), D(-6,4)$

$x_1 = 4,\ y_1 = 5,\ x_2 = -6,\ y_2 = 4$

Slope of $CD = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_2 = \dfrac{4-5}{-6-(4)} = \dfrac{-1}{-2} = \dfrac{1}{2}$

$m_1 = m_2$

$\dfrac{-9}{k-7} = \dfrac{1}{2}$

$-9 \times 2 = k - 7$

$-18 + 7 = k$

$k = -11$

Q.4 **Using slopes, show that the triangle with its vertices $A(6,1), B(2,7)$ and $C(-6,-7)$ is a right triangle.**

(i) Parallel

(ii) Perpendicular

Ans:

$A(6,1), B(2,7)$

$x_1 = 6,\ y_1 = 1,\ x_2 = 2,\ y_2 = 7$

Slope of $AB = m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \dfrac{7-1}{2-6} = \dfrac{6}{4} = \dfrac{-3}{2}$

$B(2,7), C(-6,-7)$

$x_1 = 2,\ y_1 = 7,\ x_2 = -6,\ y_2 = 7$

$m_2 = \dfrac{-7-7}{-6-2} = \dfrac{-14}{-8} = \dfrac{7}{4}$

$m_2 = \dfrac{7}{2}$

$m_1 \neq m_2$

So they are not parallel

$m_1 m_2 = -1$

$\left[\dfrac{-3}{2}\right]\left[\dfrac{2}{3}\right] = -1$

$-1 = 1$

Side $\overline{AB} \perp$ side $\overline{AC}$

ΔAB is a right angle triangle

Q.5 **Two pairs of points are given. Find whether the two lines determined by these points are:**

(i) Parallel

(ii) Perpendicular

(iii) None

(a) $(1,-2),(2,4)$ **and** $(4,1),(-8,2)$

Ans:

$(1,-2),(2,4)$

$x_1 = 1,\ y_1 = -2,\ x_2 = 2,\ y_2 = 4$

$m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \dfrac{4-(-2)}{2-1} = \dfrac{4+2}{1} = 6$

$m_2 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$(4,1),(-8,2)$

$x_1 = 4,\ y_1 = 1,\ x_2 = -8,\ y_2 = 2$

$m_2 = \dfrac{2-1}{-8-4} = \dfrac{-1}{-12}$

$m_2 = \dfrac{1}{12}$

$m_2 = 6$

(i) $m_1 \neq m_2$

Are not parallel

(ii) $m_1 m_2 = -1$

$[6]\left[\dfrac{-1}{12}\right] = -1$

$\dfrac{-1}{2} = -1$

Are not perpendicular

(iii) hence neither parallel nor perpendicular

(b) $(-3,4),(6,2)$ **and** $(4,5),(-2,-7)$

Ans:

$(-3,4)(6,2)$

$x_1 = -3,\ y_1 = 4,\ x_2 = 6,\ y_2 = 2$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{2-4}{6-(-3)} = \frac{-2}{6+3}$$

$$m_1 = \frac{-2}{9}$$

$(4,5),(-2,-7)$

$x_1 = 4,\ y_1 = 5,\ x_2 = -2,\ y_2 = -7$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_2 = \frac{-7-5}{-2-4} = \frac{-12}{-6} = 2$$

$m_2 = 2$

(i) $m_1 \neq m_2$

Not parallel

(ii) $m_1 m_2 = -1$

$$\left(\frac{-2}{3}\right)(2) = -1$$

$$\frac{-4}{3} = -1$$

Not perpendicular

(iii) Neither parallel nor perpendicular

Q.6 Find an equation of:

(a) The horizontal line through $(7,-9)$

Ans:

$x_1 = 7 \quad y_1 = 9$

$y - y_1 = m(x - x_1)$

$y - (-9) = m(x - 7)$

$y + 9 = 0(x - 7)$

$y + 9 = 0$

(b) The vertical line through $(-5,3)$

Ans:

$(x_1, y_1) = (-5,3)$

$y - y_1 = m(x - x_1)$

$0(y - 3) = 1(x + 5)$

$0 = x + 5$

$x + 5 = 0$

(c) Through $A(-6,5)$ having slope 7

Ans:

$x_1 = -6,\ y_1 = 5,\ m = 7$

$y - y_1 = m(x - x_1)$

$y - 5 = 7(x - (-6))$

$y - 5 = 7x + 42$

$= 7x - y + 42 + 5$

$7x - y + 47 = 0$

(d) Through $(8,-3)$ having slope 0

Ans:

$x_1 = 8,\ y_1 = -3,\ m = 0$

$y - y_1 = m(x - x_1)$

$y - (-3) = 0(x - 8)$

$y + 3 = 0$

(e) Through $(-8,5)$ having slope undefined

Ans:

$x_1 = -8,\ y_1 = 5,\ m = \alpha$

$y - y_1 = m(x - x_1)$

$y - 5 = \alpha(x - (-8))$

$$y - 5 = \frac{1}{6}(x + 8)$$

$6(y - 5) = x + 8$

$0 = x + 8$

$x + 8 = 0$

(f) Through $(-5,-3)$ and $(9,-1)$

Ans:

$x_1 = -5,\ y_1 = -3,\ x_2 = 9,\ y_2 = -1$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-(-3)}{-1-(-3)} = \frac{x-(-5)}{9-(-5)}$$

$$\frac{y+3}{-1+3} = \frac{x+5}{9+5}$$

$$\frac{y+3}{2} = \frac{x+4}{14}$$

$$y + 3 = 2\frac{(x+14)}{14}$$

$$\frac{y+3}{1} = \frac{x+5}{7}$$

$y(y+3)=x+5$

$7y+21=x+5$

$0=x-7y+5-21$

$x-7y-16=0$

(g) *y*-intercept: –7 and slope: –5

Ans:

y-intercept –7 and slope –5

$m=-5, c=-7$

$y=mn+c$

$y=-5x-7$

$y+5x-7$

$y+5x+7=0$

$5x+y+7=0$

(h) *x*-intercept –3 and *y*-intercept 4

Ans:

$4x-3y+12=0$

$\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{-3}+\frac{4}{4}=1$

$\frac{-x}{3}+\frac{4}{4}=1$

$\frac{-4x+3y}{12}=1$

$-4x+3y$

$43-3y+12=0$

(i) *x*-intercept: –9 and slope: –4

Ans:

$(x_1,y_1)=(-9,0)\ m=-4$

$y-y_1=m(x-x_1)$

$y-0=-4(x-(-9))$

$y=-4x-36$

$4x-y+36=0$

Q.7 Find an equation of the perpendicular bisector of the segment joining the pints $A(3,5)$ and $B(9,8)$

Ans:

$m_1=\frac{y_2-y_1}{x_2-x_1}$

$m_1=\frac{8-5}{9-3}=\frac{3}{6}=\frac{1}{2}$

Slope required $=m_1m=-1$

$m_1m=-1$

$m=\frac{-1}{m_1}$

$m=-1\div\frac{1}{2}$

$m=-1\times\frac{2}{1}$

$m=-2$

Midpoint A and $B=\left(\frac{3+9}{2},\frac{5+8}{2}\right)$

$=\left(\frac{12}{2},\frac{13}{2}\right)=\left(6,\frac{13}{2}\right)$

$y_1-y_1=m(x-x_1)$

$y-\frac{13}{2}=-2(x-6)$

$\frac{2y-13}{2}=-2x+12$

$2y-13=-4x+24$

$4x+2y-13-24=0$

$4x-2y-37=0$

Q.8 Find an equation of the line through $(-4,-6)$ and perpendicular to a line having slope $\frac{-3}{2}$.

Ans:

Slope of required line $=m=-\left(\frac{1}{m}\right)$

$=-\left(\frac{-1}{-\frac{3}{2}}\right)=-\left(\frac{-2}{3}\right)=\frac{2}{3}$

$y-y_1=m(x-x_1)$

$[y-(-0)]=\frac{2}{3}(x-(-4))$

$y+6=\frac{2}{3}(x+4)$

$3y+18=2x+8$

$=2x-3y-18+8$

$= 2x - 3y - 18 + 8$

$2x - 3y - 10 = 0$

Q.9 **Find an equation of the line through $(11, -5)$ and parallel to a line with slope –24**

Ans:

$x_1 = 11,\ y_1 = 5$

$y - y_1 = m(x - x_1)$

$y - (-5) = -24(x - 11)$

$y + 5 = -24x + 264$

$24x + y + 5 - 264 = 0$

$24x + y - 259 = 0$

Q.10 **Convert each of the following equations into slope intercept form, two intercept form and normal form:**

Ans:

(a) $2x - 4y + 11 = 0$

Slope intercept form $(y = mx + c)$

$2x - 4y + 11 = 0$

$2x + 11 = 4y$

$\frac{2}{4}x + \frac{11}{4} = y$

$m = \frac{1}{2},\ c = \frac{11}{4}$

(ii) Two intercept form $\left[\frac{x}{a} + \frac{y}{b} = 1\right]$

$2x - 4y = -11$

Dividing both side by –11

$\frac{2x}{-11} - \frac{4y}{-11} = \frac{-11}{-11}$

$\frac{-2x}{11} + \frac{4}{11}y = 1$

$\frac{x}{\frac{-11}{2}} + \frac{y}{\frac{11}{4}} = 1$

$a = \frac{-11}{2}$ $\quad$ $b = \frac{11}{4}$

(iii) Normal form

$(x\cos\alpha + y\sin\alpha = p)$

$2x - 4y = -11$

$-2x + 4y = 11$

Divide both side by $\sqrt{(-2)^2 + (4)^2}$

$= \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

Equation becomes

$\frac{-2}{2\sqrt{5}}x + \frac{4}{2\sqrt{5}}y = \frac{11}{2\sqrt{5}}$

$\frac{-1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = \frac{11}{2\sqrt{5}}$

$\cos\alpha = \frac{-1}{\sqrt{5}}$ $\quad$ $\sin\alpha = \frac{2}{\sqrt{5}}$

$y = \frac{11}{2\sqrt{5}}$

(b) $4x + 7y - 2 = 0$

(i) Slope intercept form $y = mx + c$

$7y = -4x + 2$

$y = \frac{-4}{7}x + \frac{2}{7}$

$m = \frac{-4}{7}$ $\quad$ $c = \frac{2}{7}$

(ii) Two intercept form $\left[\frac{x}{a} + \frac{y}{b} = 1\right]$

$4x + 7y = 2$

Dividing by 2 whole equal

$\frac{4x}{2} + \frac{7y}{2} = \frac{2}{2}$

$2x + \frac{7}{2}y = 1$

$\frac{x}{\frac{1}{2}} + \frac{y}{\frac{2}{7}} = 1$

$a = \frac{1}{2}$ $\quad$ $b = \frac{2}{7}$

(iii) Normal form

$(x\cos\alpha + y\sin\alpha = p)$

$4x + 7y = 2$

Dividing

$\sqrt{(4)^2 + (7)^2} = \sqrt{16 + 49} = \sqrt{65}$

Equation becomes $\frac{4x}{\sqrt{65}} + \frac{7y}{\sqrt{65}} = \frac{2}{\sqrt{65}}$

Which is normal form with

$\cos\alpha = \frac{4}{\sqrt{65}}$

$\sin\alpha = \frac{7}{\sqrt{65}}$ $\quad$ $p = \frac{2}{\sqrt{65}}$

Length of perpendicular form $(0,0)$

to line $4x+7y-2=0$ is $\frac{2}{\sqrt{65}}$

(c) $-8x+15y+3=0$

(i) Slope intercept form $y=mx+c$

$15y=8x-3$

$y=\frac{8x}{15}-\frac{3}{15}$

$y=\frac{8}{15}x-\frac{1}{5}$

$m=\frac{8}{15}$ $\quad c=\frac{-1}{5}$

(ii) Two intercept form $\left[\frac{x}{a}+\frac{y}{b}\right]=1$

$-8x+15y=-3$

Dividing equation by –3

$\frac{-8x}{-3}+\frac{15y}{-3}=\frac{-3}{-3}$

$\frac{8x}{3}-5y=1$

$\frac{8}{3}x-5y=1$

$\frac{x}{\frac{3}{8}}+\frac{y}{\frac{-1}{5}}=1$

$a=\frac{3}{8}$ $\quad b=\frac{-1}{5}$

Q.11 In each of the following check whether the two lines are

(i) Parallel

(ii) Perpendicular

(iii) Neither parallel nor perpendicular

Ans:

(a) $2x+y-3=0;\quad 4x+2y+5=0$

$\ell_1=2x+y-3=0$

$\ell_2=4x+2y+5=0$

$a=2,\quad b=1\quad c=-3$

$m_1=\ell_1=\frac{-a}{b}$

$m_1=\frac{-2}{1}=-2$

$m_2=\ell_2=\frac{-a}{b}$

$=\frac{-4}{2}=-2$

$m_1=m_2=-2$

$m_1m_2=-1$

$(-2)(-2)=-1$

$4=-1$

Lines are parallel only

(b) $3y=2x+5;\quad 3x+2y-8=0$

Ans:

$\ell_1=3y=2x+5$

$\ell_2=3x+2y-8$

$a=2,b=-3,c=5$

$m_1=\ell_1=\frac{-a}{b}=\frac{-2}{-3}$

$=\frac{2}{3}$

$m_2=\ell_2=\frac{-a}{b}=\frac{-3}{2}$

$m_1m_1=-1$

$\left[\frac{2}{3}\right]\left[\frac{-3}{2}\right]=-1$

$-1=1$

(c) $4y+2x-1=0;\quad x-2y-7=0$

Ans:

$\ell_1=2x+4y-1=0$

$a=2,b=4,c=-1$

$\ell_2=x-2y-7=0$

$a=1,b=-2,c=-7$

$\ell_1=m_1=\frac{-a}{b}=\frac{-2}{-4}$

$=\frac{-1}{2}$

$\ell_2=m_2=\frac{-a}{b}=\frac{-1}{-2}=\frac{1}{2}$

Given lines are neither parallel or perpendicular

Q.12 Find an equation of the line through $(-4,7)$ and parallel to the line $2x-7y+4=0$

Ans:

Given parallel line $2x-7y+4=0$

Slope $=\frac{-a}{b}$

$a=2,\ b=-7,\ c=4$

Slope $=\frac{-2}{-7}=\frac{2}{7}$

Slope of required line $=\frac{2}{7}$

As lines are parallel so slope are equal

Required equation

$y-y_1=m(x-x_1)$

$y-7=\frac{2}{7}(x+4)$

$2x-7y+49+8=0$

$2x-7y+57=0$

Q.13 Find an equation of the line through $(5,-8)$ and perpendicular to the join of $A(-15,-8), B(10,7)$

Ans:

Slope of $AB=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{7+8}{10-(-15)}=\frac{15}{25}=\frac{3}{5}$

So slope of required perpendicular line

$=\frac{-5}{3}$

$m=\frac{3}{5}$

$m_1m'=-1$

$m'=\frac{-1}{m}$

$m'=\frac{-1}{\frac{3}{5}}$

$m'=\frac{-5}{3}$

Equation of required line

$y-y_1=m(x-x_1)$

$y(-8)=\frac{-5(x-5)}{3}$

$y+8=\frac{-5}{3}(x-5)$

$3y+24=-5x+25$

$5x+3y+24-25=0$

$5x+3y-1=0$