REVIEW EXERCISE

Q.1 Four options are given against each statement. Encircle the correct option.

(i) The set builder from of the set $\left\{1,\frac{1}{3},\frac{1}{5},\frac{1}{7},...\right\}$ is:

(a) $\left\{x \mid x=\frac{1}{n}, n \in W\right\}$

(b) $\left\{x \mid x=\frac{1}{2n+1}, n \in W\right\}$

(c) $\left\{x \mid x=\frac{1}{n+1}, n \in W\right\}$

(d) $\{x \mid x=2n+1, n \in W\}$

(ii) If $A=\{\ \}$, **then** $P(A)$ **is:**

(a) $\{\ \}$ (b) $\{1\}$

(c) $\{\{\ \}\}$ (d) ϕ

(iii) If $U=\{1,2,3,4,5\}, A=\{1,2,3\}$ **and** $B=\{3,4,5\}$, **then** $U-(A \cap B)$ **is:**

(a) $\{1,2,4,5\}$ (b) $\{2,3\}$

(c) $\{1,3,4,5\}$ (d) $\{1,2,3\}$

(iv) If A **and** B **are overlapping sets, then** $n(A-B)$ **is equal to**

(a) $n(A)$ (b) $n(B)$

(c) $A \cap B$

(d) $n(A)-n(A \cap B)$

(v) If $A \subseteq B$ **and** $B-A \neq \phi$ **then** $n(B-A)$ **is equal to**

(a) 0 (b) $n(B)$

(c) $n(A)$

(d) $n(B)-n(A)$

(vi) If $n(A \cup B)=50, n(A)=30$ **and** $n(B)=35$, **then** $n(A \cap B)=$:

(a) 23 (b) 15

(c) 9 (d) 40

(vii) If $A=\{1,2,3,4\}$ **and** $B=\{x,y,z\}$, **then cartesian product of** A **and** B **contains exactly _____ elements.**

(a) 13 (b) 12

(c) 10 (d) 6

(viii) If $f(x)=x^2-3x+2$, **then the value of** $f(a+1)$ **is equal to:**

(a) $a+1$ (b) a^2+1

(c) a^2+2a+1 (d) a^2-a

(ix) Given that $f(x)=3x+1$, **if** $f(x)=28$, **then the value of** x **is:**

(a) 9 (b) 27

(c) 3 (d) 18

(x) Let $A=\{1,2,3\}$ **and** $B=\{a,b\}$ **two non-empty sets and** $f: A \to B$ **be a function defined as** $f=\{(1,a),(2,b),(3,b)\}$, **then which of the following statement is true?**

(a) f is injective

(b) f is surjective

(c) f is bijective (d) f is into only

Answer Key

1	b	**2**	c	**3**	a	**4**	d	**5**	d
6	b	**7**	b	**8**	d	**9**	a	**10**	b

Q.2 Write each of the following sets in tabular forms:

(i) $\{x \mid x=2n, n \in N\}$

Ans:

$=\{2(1),2(2),2(3),...\}$

$=\{2,4,6,...\}$

(ii) $\{x \mid x=2m+1, m \in N\}$

Ans:

$=\{2(1)+1,2(2)+1,2(3)+1,...\}$

$=\{3,5,7,...\}$

(iii) $\{x \mid x=11n, n \in W \wedge n<11\}$

Ans:

$=\{11(0),11(1),11(2),11(3),...11(10)\}$

$=\{0,11,22,33,...110\}$

(iv) $\{x \mid x \in E \wedge 4<x<6\}$

Ans:

$= \{\ \}$

(v) $\{x \mid x \in O \wedge 5 \leq x < 7\}$

Ans:

$= \{5\}$

(vi) $\{x \mid x \in Q \wedge x^2 = 2\}$

Ans:

$= x^2 = 2$

$\sqrt{x^2} = \sqrt{2}$

$x = \sqrt{2}$

$\sqrt{2}$ is irrational $\in Q'$

So $\{\ \}$

(vii) $\{x \mid x \in Q \wedge x = -x\}$

Ans:

$3 = -3$

$\frac{1}{9} = \frac{-1}{9}$

$\{0\}$

(viii) $\{x \mid x \in R \wedge x \notin Q\}$

Ans:

$\{Q\}$

Q.3 **Let** $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{2,4,6,8,10\}, B = \{1,2,3,4,5\}$ **and** $C = \{1,3,5,7,9\}$

List the members of each of the following sets:

(i) A'

Ans:

$A' = U - A$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$

$= \{1,3,5,7,9\}$

(ii) B'

Ans:

$B' = U - B$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5\}$

$= \{6,7,8,9,10\}$

(iii) $A \cup B$

Ans:

$A \cup B = \{2,4,6,8,10\} \cup \{1,2,3,4,5\}$

$A \cup B = \{1,2,3,4,5,6,8,10\}$

(iv) $A - B$

Ans:

$A - B = \{2,4,6,8,10\} - \{1,2,3,4,5\}$

$= \{6,8,10\}$

(v) $A \cap C$

Ans:

$A \cap C = \{2,4,6,8,10\} \cap \{1,3,5,7,9\}$

$= \{\ \}$

(vi) $A' \cup C'$

Ans:

$A' = \{1,3,5,7,9\}$

$C' = U - C$

$= \{1,2,3,4,5,6,7,9,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$A' \cup C' = \{1,3,5,7,9\} \cup \{2,4,6,8,10\}$

$A' \cup C' = \{1,2,3,4,5,6,7,8,9,10\}$

(vii) $A' \cup C$

Ans:

$A' = \{1,3,5,7,9\}, C = \{1,3,5,7,9\}$

$A' \cup C = \{1,3,5,7,9\} \cup \{1,3,5,7,9\}$

$A' \cup C = \{1,3,5,7,9\}$

(viii) U'

Ans:

$U' = U - U$

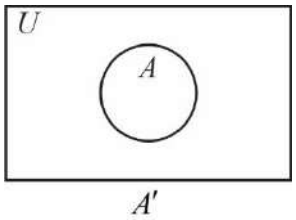
$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5,6,7,8,9,10\}$

$= \{\ \}$

Q.4 **Using the Venn diagrams, if necessary, find the single sets equal to the following:**

(i) A'

Ans:



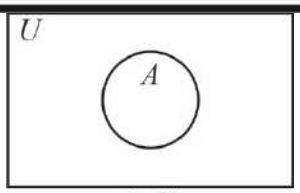
A'

$A' = U - A$

$A' = A$

(ii) $A \cap B$

Ans:



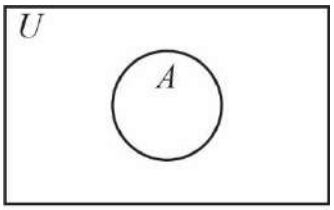
$A \cap U$

$A \cap U = A$

(iii) $A \cup U$

Ans:

$A \cup U = U$

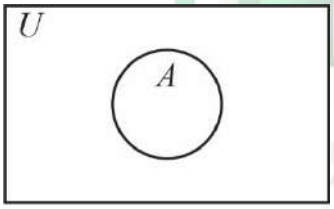


$A \cup U$

(iv) $A \cup \phi$

Ans:

$A \cup \phi = A$



$A \cup \phi$

(v) $\phi \cup \phi$

Ans:

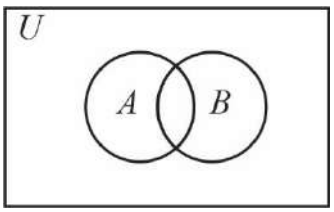
$\phi \cap \phi = \phi$

Q.5 **Use Venn diagrams to verify the following:**

(i) $A - B = A \cup B'$

Ans:

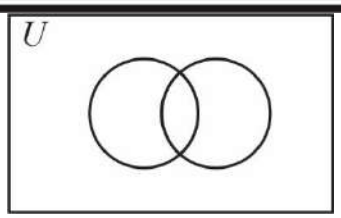
$A - B = A \cup B'$



(ii) $(A - B)' \cap B = B$

Ans:

$(A - B)' \cap B = B$



Q.6 **Verify the properties for the sets A, B and C given below:**

(i) **Associativity of Union**

Ans:

$A \cup (B \cup C) = (A \cup B) \cup C$

(a) $A = \{1,2,3,4\}$, $B = \{3,4,5,6,7,8\}$,

$C = \{5,6,7,9,10\}$

L.H.S $A \cup (B \cup C)$

$B \cup C = \{3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$

$B \cup C = \{3,4,5,6,7,8,9,10\}$

$A \cup (B \cup C) = \{1,2,3,4\} \cup \{3,4,5,6,7,8,9,10\}$

$A \cup (B \cup C) = \{1,2,3,4,5,6,7,8,9,10\}$

R.H.S $(A \cup B) \cup C$

$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$

$A \cup B = \{1,2,3,4,5,6,7,8\}$

$A \cup B \cup C = \{1,2,3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$

$(A \cup B) \cup C = \{1,2,3,4,5,6,7,8,9,10\}$

$A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S = R.H.S

(ii) **Associativity of intersection.**

Ans:

$A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S

$A \cap (B \cap C)$

$A \cap (B \cap C)$

$= \{1,2,3,4\} \cap [\{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}]$

$= \{1,2,3,4\} \cap \{5,6,7\} = \{\ \}$

R.H.S

$(A \cap B) \cap C$

$(A \cap B) \cap C$

$= [\{1,2,3,4\} \cap \{3,4,5,6,7,8\} \cap \{5,6,7,9\}]$

$= \{3,4\} \cap \{5,6,7,9,10\} = \{\ \}$

Hence prove L.H.S = R.H.S

(iii) Distributivity of Union over intersection.

Ans:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $A \cup (B \cap C)$

$A \cup (B \cap C)$

$= \{1,2,3,4\} \cup \{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}$

$= \{1,2,3,4\} \cup \{5,6,7\}$

$= \{1,2,3,4,5,6,7\}$

R.H.S $(A \cup B) \cap (A \cup C)$

$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$

$A \cup B = \{1,2,3,4,5,6,7,8\}$

$A \cup C = \{1,2,3,4\} \cup \{5,6,7,9,10\}$

$A \cup C = \{1,2,3,4,5,6,7,9,10\}$

$(A \cup B) \cap (A \cup C)$

$= \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,4,5,6,7,9,10\}$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6,7\}$

L.H.S = R.H.S

(iv) Distributivity of intersection over union.

Ans:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $A \cap (B \cup C)$

$A \cap (B \cup C)$

$= \{1,2,3,4\} \cap \{3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$

$A \cap (B \cup C)$

$= \{1,2,3,4\} \cap \{3,4,5,6,7,8,9,10\}$

$A \cap (B \cup C) = \{3,4\}$

R.H.S

$(A \cap B) \cup (A \cap C)$

$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6,7,8\}$

$A \cap B = \{3,4\}$

$A \cap B = \{1,2,3,4\} \cap \{5,6,7,9,10\}$

$A \cap B = \{\ \}$

$(A \cap B) \cup (A \cap C) = \{3,4\} \cup \{\ \}$

$= \{3,4\}$

(b) $A = \phi, B = \{0\}, C = \{0,1,2\}$

Ans:

(i) $A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S

$A \cup (B \cup C)$

$A \cup (B \cup C) = \{\ \} \cup (\{0\} \cup \{0,1,2\})$

$= \{\ \} \cup \{0,1,2\} = \{0,1,2\}$

$(A \cup B) \cup C = [\{\ \} \cup \{0\}] \cup \{0,1,2\}$

$= \{0\} \cup \{0,1,2\}$

$= \{0,1,2\}$

Hence prove L.H.S = R.H.S

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S

$A \cap (B \cap C) = \{\ \} \cap (\{0\} \cap \{0,1,2\})$

$A \cap (B \cap C) = \{\ \} \cap \{0\}$

$A \cap (B \cap C) = \{\ \}$

R.H.S

$(A \cap B) \cap C$

$(A \cap B) \cap C = (\{\ \} \cap \{0\}) \cap \{0,1,2\}$

$= \{\ \} \cap \{0,1,2\} = \{\ \}$

Hence proved L.H.S = R.H.S

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S

$A \cup (B \cap C) = \{\ \} \cup (\{0\} \cap \{0,1,2\})$

$= \{\ \} \cup \{0\} = \{0\}$

R.H.S

$(A \cup B) \cap (A \cup C)$

$A \cup B = \{\ \} \cup \{0\}$

$A \cup B = \{0\}$

$A \cup C = \{0\} \cup \{0,1,2\}$

$A \cup C = \{0,1,2\}$

$(A \cup B) \cap (A \cup C) = \{0\} \cap \{0,1,2\}$

$= \{0\}$

Hence proved L.H.S = R.H.S

$A \cap (B \cup C) = (A \cap B) \cup \{A \cap C\}$

L.H.S

$A \cap (B \cup C)$

$A \cap (B \cup C) = \{\ \} \cap (\{0\} \cup \{0,1,2\})$

$= \{\ \} \cap \{0,1,2\} = \{\ \}$

R.H.S

$(A \cap B) \cup (A \cap C)$

$A \cap B = \{\ \} \cap \{0\} = \{\ \}$

$A \cap C = \{\ \} \cap \{0,1,2\}$

$A \cap C = \{\ \}$

$(A \cap B) \cup (A \cap C) = \{\ \} \cup \{\ \} = \{\ \}$

Hence proved L.H.S = R.H.S

(c) $A = N, B = Z, C = Q$

Ans:

$A = N = \{1,2,3,...\}, B\{0, \pm 1, \pm 2, ...\}$

$A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S

$A \cup (B \cup C) = \{1,2,3,...\} \cup \{0, \pm 1, \pm 2, \pm 3, ...\} \cup Q$

$A \cup (B \cup C) = \{1,2,3,...\} \cup Q = Q$

R.H.S $(A \cup B) \cup C$

$(A \cup B) \cup C = \{1,2,3,...\} \cup \{0, \pm 1, \pm 2, ...\} \cup Q$

$= \{0, \pm 1, \pm 2, ...\} \cup Q = Q$

Hence proved L.H.S = R.H.S

Q.7 Verify De Morgan's laws for the following sets:

$U = \{1,2,3,...,20\}, A = \{2,4,6,...,20\}$

and $B = \{1,3,5,...,19\}$

Ans:

(i) $(A \cup B)' = A' \cap B'$

L.H.S

$(A \cup B)'$

$A \cup B = \{2,4,6,8,...,20\} \cup \{1,3,5,7,...,19\}$

$A \cup B = \{1,2,3,4,5,...,20\} \cup \{1,3,5,7,...,19\}$

$(A \cup B)' = U - (A \cup B)$

$= \{1,2,3,...,20\} - \{1,2,3,4,...,20\} = \{\ \}$

R.H.S

$A' \cap B'$

$A' = U - A$

$A' = \{1,2,3,...,20\} - \{2,4,6,...,20\}$

$B' = U - B$

$B' = \{1,2,3,...,20\} - \{1,3,5,7,...,19\}$

$B' = \{2,4,6,8,...,20\}$

$A' \cap B' = \{\ \}$

Hence prove L.H.S = R.H.S

(ii) $(A \cap B)' = (A' \cup B')$

L.HS

$(A \cap B)'$

$A \cap B = \{2,4,6,8,...,20\} \cap \{1,3,5,7,...,19\}$

$A \cap B = \{\ \}$

$(A \cap B)' = U - (A \cap B)$

$= \{1,2,3,...,20\} - \{\ \} = \{1,2,3,...,20\}$

R.H.S

$A' \cup B'$

$A' = U - A$

$A' = \{1,2,3,...,20\} - \{2,4,6,...,20\}$

$A' = \{1,3,5,7,...,19\}$

$B' = U - B$

$B' = \{1,2,3,...,20\} - \{1,3,5,7,...,19\}$

$B' = \{2,4,6,8,...,20\}$

$A' \cup B' = \{1,3,5,...,19\} \cup \{2,4,6,8,...,20\}$

$A' \cup B' = \{1,2,3,4,5,...,20\}$

Hence proved L.H.S = R.H.S

Q.8 **Consider the set**

$P=\{x \mid x=5m, m \in N\}$ **and**

$Q=\{x \mid x=2m, m \in N\}$. **find** $P \cap Q$

Ans:

$P=\{5(1),5(2),5(3),...\}$

$P=\{5,10,15,...\}$

$Q=\{2(1),2(2),2(3),...\}$

$Q=\{2,4,6,...\}$

$P \cap Q=\{5,10,15,...\} \cap \{2,4,6,...\}$

$P \cap Q=\{10,20,30,40,...\}$

Q.9 **From suitable properties of union and intersection, deduce the following results:**

(i) $A \cap (A \cup B)=A \cup (A \cap B)$

Ans:

L.H.S

$A \cap (A \cup B)$

$(A \cap A) \cup (A \cup B)$

Since $A \cap A=A$

$A \cap (A \cup B)=A \cup (A \cap B)$

(ii) $A \cup (A \cap B)=A \cap (A \cup B)$

Ans:

L.H.S

$A \cup (A \cap B)$

$(A \cup A) \cap (A \cup B)$

$A \cup A=A$

$A \cap (A \cup B)$

L.H.S = R.H.S

Q.10 **If** $g(x)=7x-2$ **and** $s(x)=8x^2-3$ **find:**

(i) $g(0)$

Ans:

$g(0)=7(0)-2$

$g(0)=-2$

(ii) $g(-1)$

Ans:

$g(-1)=7(-1)-2$

$=-7-2=-9$

(iii) $g\left(-\frac{5}{3}\right)$

Ans:

$g\left(\frac{-5}{3}\right)=7\left(\frac{-5}{3}\right)-2$

$=\frac{-35}{3}-2=\frac{-35-6}{3}=-\frac{4}{3}$

(iv) $s(1)$

Ans:

$sx=8x^2-3$

$s(1)=8(1)^2-3$

$=8-3=5$

(v) $s(-9)$

Ans:

$s(-9)=8(-9)^2-3$

$=8(81)=648-3=645$

(vi) $s\left(\frac{7}{2}\right)$

Ans:

$s\left(\frac{7}{2}\right)=8\left[\frac{7}{2}\right]^2-3$

$=8\left[\frac{49}{4}\right]-3=2(49)-3=98-3=95$

Q.11 **Given that** $f(x)=ax+b$, **where** a **and** b **are constant numbers. If** $f(-2)=3$ **and** $f(4)=10$, **, then find the values of** a **and** b.

Ans:

$f(x)=ax+b$

$f(-2)=a(-2)+b$

$f(-2)=-2a+b$

$f(2)=3$

$3=-2a+b$

$-2a+b=3$ _____(i)

$f(4)=a(4)+b$

$f(4)=40+b$

$10=4a+b$

$40+b=10$ _____(ii)

Subtracting equation (i) from (ii)

$4a+b=10$

$\mp 2a \pm b=\pm 3$

$6a=7$

$a = \frac{7}{6}$

Putting the value of a in equation (i)

$-2\left(\frac{7}{6}\right) + b = 3$

$-\frac{7}{3} + b = 3$

$b = 3 + \frac{7}{3}$

$b = \frac{9+7}{3}$

$b = \frac{16}{3}$

Q.12 Consider the function defined by $k(x) = 7x - 5$. If $k(x) = 100$, find the value of x.

Ans:

$K(x) = 7x - 5$

$100 = 7x - 5$

$100 + 5 = 7x$

$7x = 105$

$x = \frac{105}{7}$

$x = 15$

Q.13 Consider the function $g(x) = mx^2 + n$, where m and n are constant numbers. If $g(4) = 20$ and $g(0) = 5$, find the values of m and n.

Ans:

$g(x) = mx^2 + n$

$g(4) = mx^2 + n$

$g(4) = m(4)^2 + n$

$20 = 16m + n$

$16m + n = 20$ ____(i)

$g(0) = m(0)^2 + n$

$5 = n$

$n = 5$

Putting the value of n in equation (1)

$16m + 5 = 20$

$16m = 20 - 5$

$16m = 15$

$m = \frac{15}{16}$

Q.14 A shopping mall has 100 products from various categories labeled 1 to 100, representing the universal set U. the products are categorized as follows:

- **Set A: Electronics, consisting of 30 products labeled from 1 to 30.**
- **Set B: Clothing comprises 25 products labeled from 31 to 55.**
- **Set C: Beauty products, comprising 25 products labeled from 76 to 100.**

Write each set in tabular form, and inf the union of all three sets.

Ans:

$A = \{1, 2, 3, ..., 10\}$

Clothing $B = \{31, 32, 33, ..., 55\}$

Beauty product

$C = \{76, 77, 78, 79, 80, ..., 100\}$

$A \cup B \cup C = \{1, 2, 3, ...,\} \cup \{31, 32, ...,\} \cup \{76, 77, ..., 100\}$

$A \cup B \cup C = \{1, 2, 3, ..., 100\}$

Q.15 Out of the 180 students who appeared in the annual examination, 120 passed the math test, 90 passed the science test, and 60 passed both the math and science tests.

(a) How many passed either the math or science test?

(b) How many did not pass either of the two tests?

(c) How many passed the science test but not the math test?

(d) How many failed the science test?

Ans:

(a)

Maths Students $n(M) = 120$

Science students $n(S)=90$

$n(M\cap S)=60$

$n(M\cup S)=?$

$n(M\cup S)=n(M)+n(S)=n(M\cap S)$

$=120+90-60=150$

(b)

Total students

$n(M\cup S)=180-150=30$

Students which are failed = 30

(c)

Students passed the science test but not the math test

$=n(S)-n(M\cap S)=90-60=30$

(d)

Students fail science test $=n(S)$

Total $-n(S)$

$=180-90=90$

Q.16 In a software house of a city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survey revealed the following statistics:

- **150 developers like Python.**
- **130 developers like Java.**
- **120 developers like PHP.**
- **70 developers like both Python and Java.**
- **60 developers like both Python and PHP.**
- **50 developers like both Java and PHP.**
- **40 developers like all three languages: Python, Java and PHP.**

(a) How many developers use at least one of these languages?

Ans:

$n(P)=150$

$n(J)=130$

$n(H)=120$

$n(P\cap J)=70$

$n(P\cap H)=60$

$n(J\cap H)=50$

$n(P\cap J\cap H)=40$

$n(P\cap J\cap H)=?$

$n(P\cup J\cup H)$

$=n(P)+n(J)+n(H)-n(P\cap J)+n(P\cap J\cap H)$

$=150+130+120-70-60-50+40$

$=260$

(b) How many developers use only one of these languages?

Ans:

$=n(P)-\left[n(P\cap J)+n(P\cap H)-n(P\cap J\cap H)\right]$

$=150-\left[70+60-40\right]=150-90=60$

$n(J)-\left[n(P\cap J)+n(P\cap H)-n(P\cap J\cap PH)\right]$

$=130-\left[70+50-40\right]=130-80=50$

PHP only

$n(PH)-\left[n(P\cap PH)+n(J\cap PH)-n(P\cap J\cap PH)\right]$

$=120-\left[60+50-40\right]=120-70=50$

Developer use only language

$=60+50+50=160$

(c) How many developers do not use any of these languages?

Ans:

Does not use language = Total $-n(J\cup P\cup PH)$

$=300-260=40$

(d) How many developers use only PHP?

Ans:

$=50$