Thus, $(x+1)^2+7=x^2+2x+8.$ Hence proved.

REVIEW EXERCISE

Q.1 Four options are given against each statement. Encircle the correct option.

(i) Which of the following expressions is often related to inductive reasoning?
(a) Based on repeated experiments
(b) If an only if statements
(c) Statement is proven by a theorem
(d) based on general principles

(ii) Which of the following sentences describe deductive reasoning?
(a) General conclusions from a limited number of observations
(b) Base on repeated experiments
(c) Based on units of information that are accurate
(d) Draw conclusion from well-known facts

(iii) Which one of the following statements is true?
(a) The set of integers is finite
(b) The sum of the interior angles of any quadrilateral is always 80°
(c) $\frac{22}{7}\notin Q'$
(d) All isosceles triangles are equilateral triangles

(iv) Which of the following statements is the best to represent the negation of the statement "The stove is burning"?
(a) The stove is not burning
(b) The stove is dim
(c) The stove is turned to low heat
(d) it is both burning and not burning

(v) The conjunction of two statements p and q is true when
(a) both p and q are false
(b) both p and q are true
(c) only q is true
(d) only p is true

(vi) A conditional is regarded as false only when:
(a) Antecedent is true and consequent is false.
(b) consequent is true and antecedent is false
(c) antecedent is true only
(d) consequent is false only

(vii) Contrapositive of $q\to p$ is
(a) $q\to\sim p$ (b) $\sim q\to p$
(c) $\sim p\to\sim q$ (d) $\sim q\to\sim p$

(viii) The statement "Every integer greater than 32 is a sum of two prime number" is:
(a) Theorem (b) Conjecture
(c) Axiom (d) Postulates

(ix) The statement "A straight line can be drawn between any two points" is:
(a) Theorem (b) Conjecture
(c) Axiom (d) Logic

(x) The statement "The sum of the interior angle of a triangle is 180°" is:
(a) Converse (b) Theorem
(c) Axiom (d) Conditional

Answer Key

1	a	**2**	d	**3**	c	**4**	a	**5**	b
6	a	**7**	c	**8**	b	**9**	c	**10**	b

Q.2 Write the converse, inverse and contrapositive of the following conditionals:

(i) $\sim q\to p$

Ans:
Converse is $q\to\sim p$
Inverse is $p\to\sim q$
Contra positive is $\sim q\to p$

(ii) $q\to p$

Ans:
Converse is $q\to p$
Inverse is $\sim q\to\sim q$
Contra positive is $\sim p\to\sim q$

(iii) $\sim p\to\sim q$

Ans:
Converse is $\sim q\to\sim p$
Inverse is $p\to q$
Contra positive is $q\to p$

(iv) $\sim q \to \sim p$

Ans:

Converse is $\sim p \to \sim q$

Inverse is $q \to p$

Contra positive is $p \to q$

Q.3 Write the truth table of the following

(i) $\sim (p \vee q) \vee (\sim q)$

Ans:

p	q	$\sim q$	$p \vee q$	$\sim (p \vee q)$	$\sim (p \vee q) \vee \sim (q)$
T	T	F	T	F	F
T	F	T	T	F	T
F	T	F	T	F	F
F	F	T	F	T	T

(ii) $\sim (\sim q \vee \sim p)$

Ans:

p	q	$\sim p$	$\sim q$	$\sim q \vee \sim p$	$\sim (\sim q \vee \sim p)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

(iii) $(p \vee q) \leftrightarrow (p \wedge q)$

Ans:

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \leftrightarrow (p \wedge q)$
F	F	F	F	T
T	F	T	F	F
F	T	T	F	F
T	T	T	T	T

Q.4 Differentiate between a mathematical statement and its proof. Given two examples

Ans:

Statement

A sentence or mathematical expression which may be true or false but not both is called a statement. This is correct so far as mathematics and other sciences are concerned for instance. The statement $a = b$ can be either true or false. In statistical or social sciences. It is sometimes impossible to divide all statements into two mutually exclusive classes some statement may be for instance undecided.

Example

(i) The sum of two odd integers is an even integer.

(ii) Mathematical statement are false all isosceles triangle are equilateral

Proof

Proof is a logical and systematic demonstration that a statement known as theorem. Is true a proof is a series of logical steps that show that a theorem false necessarily and inevitably from a set of axioms.

Q.5 What is the difference between an axiom and a theorem? Give examples of each.

Ans:

An axiom is a statement that is assumed to be true without proof. Axioms are the foundation of a mathematical theory and they are used to define the basic concepts and rules of the theory axiom are often self-evident or universally accepted and they are used as the starting point for proving other statement e.g. $a = b, b = c, a = c$

Theorem on the other hand is a statement that has been proven to be true using a logical argument based on axioms and previously proven statements. Theorems are the results of mathematical proofs any they are often used to describe the properties and behavior of mathematical object

For example

$(\text{Hyp})^2 = (\text{Prep})^2 + (\text{base})^2$

$c^2 = a^2 + b^2$

This theorem has been proven to be true using logical argument based on axioms and previously proven statements and it is a fundamental result in geometry.

Q.6 What is the importance of logical reasoning in mathematical proofs? Given an example to illustrate your point.

Ans:

Reasoning is crucial aspect of mathematical proof as it enables mathematics to logically declare the truth of a statement from set of axioms
Example proof that the sum of two odd number in even
Let
$x=2k+1$_____(i)
$y=2m+1$____(ii)
Adding (i) and (ii)
$x=2k+1$
$y=2m+1$
$x+y=2k+2m+2$
$x+y=2[k+m+1]$
Since $k+m+1$ is an integer $x+y$ is even.

Q.7 Indicate whether it is an axiom, conjecture or theorem and explain your reasoning.

(i) There is exactly one straight line through any two points.

Ans:
"Through any two points, there is exactly one straight line". The is Euclid Axiom

(ii) Every even number greater than 2 can be written as the sum of two prime numbers."

Ans:
$2+2=4$
$6=2+3=5$
$10=2\times5, 2+5=7$
Theorem

(iii) The sum of the angles in a triangle is 180°

Ans:
Sum of three angles in a triangle is 180°
$\angle30°+\angle60°+\angle90°=180°$

Q.8 Formulate simple deductive proof for each of the following algebraic expressions, prove that the L.H.S is equal to the R.H.S:

(i) Prove that $(x-4)^2+9=x^2-8x+25$

Ans:
L.H.S
$(x-4)^2+9$
$=(x-4)(x-4)+9$
$=x^2-4x-4x+16+9$
$=x^2-8x+25$
Hence prove L.H.S = R.H.S

(ii) Prove that $(x+1)^2-(x-1)^2=4x$

Ans:
L.H.S
$(x+1)^2-(x-1)^2$
$=(x+1)(x+1)-[(x-1)(x-1)]$
$=x^2+x+x+1-[x^2-x-x+1]$
$=x^2+2x+1-[x^2-2x+1]$
$=x^2+2x+1-x^2+2x-1$
$=4x$
Hence proved L.H.S = R.H.S

(iii) Prove that $(x+5)^2-(x-5)^2=20x$

Ans:
L.H.S
$(x+5)^2-(x-5)^2$
$=[x+5][x+5]-[x-5][x-5]$
$=x^2+5x+5x+25-[x^2-5x-5x+25]$
$=x^2+10x+25-(x^2-10-25)$
$=x^2+10x+25-x^2+10x-25$
$=20x$
Hence proved L.H.S = R.H.S

Q.9 Prove the following by justifying each step:

(i) $\dfrac{4+16x}{4}=1+4x$

Ans:
L.H.S
$\dfrac{4+16x}{4}$
$=\dfrac{4}{4}+\dfrac{16x}{4}$
$=1+4x$
L.H.S = R.H.S

(ii) $\dfrac{6x^2+18x}{3x^2-9}=\dfrac{2x}{x-3}$

Ans:

$=\frac{6x(x+3)}{3(x^2-9)}=\frac{2x(x+3)}{x^2-3^2}$

$=\frac{2x(x+3)}{(x+3)(x-3)}$

$=\frac{2x}{x-3}$

(iii) $\frac{x^2+7x+10}{x^2-3x-10}=\frac{x+5}{x-5}$

Ans:

L.H.S

$\frac{x^2+7x+10}{x^2-3x-10}$

$=\frac{x^2+5x+2x+10}{x^2-5x+2x-10}$

$=\frac{x(x+5)+2(x+5)}{x(x-5)+2(x-5)}=\frac{(x+2)(x+5)}{(x+2)(x-5)}$

$=\frac{x+5}{x-5}$

Hence proved L.H.S = R.H.S

Q.10 Suppose x is an integer. Then x is odd, then $9x+4$ is odd.

Ans:

Let x is odd

By definition on odd integer can be written as $x=2k+1$

$9x+4=9[2k+1]+4$

$=18k+9+4=18k+13$

$=18k+12+1=2(2k+6)+1$

Since $2(9k+6)+1$ is in the form

$2m+1$ if $9x+4$ is odd then x is odd

$9x+4=2m+1$

$9x=2m+1-4$

$9x=2m-3$

$9x=\frac{2}{2}[2m-3]$

$9x=2\left[\frac{2m}{2}-\frac{3}{2}\right]$

$9x=2\left(m-\frac{3}{2}\right)$

Since $9x$ is even and 9 is odd x must be odd

Q.11 Suppose x is an integer. If x is odd, then $7x+5$ is even

Ans:

Given that x is an odd integer

Odd is written $x=2k+1$

Replace x by $2k+1$ is $7x+5$

$7x+5=7(2k+1)+5$

$7x+5=14k+7+5$

$7x+5=14k+12$

$7x+5=2(7k+6)$

As 2 $(7k+6)$ is divided by 2 $7x+5$ is even

Q.12 Prove the following statements:

(a) If x is an odd integer, then show that it x^2-4x+6 is odd.

Ans:

Let x is odd

$x=2k+1$

Replace x by $2k+1$

$4x^2-4x+6=(2k+1)-4(2k+1)+6$

$=4k^2+4k+1-8k-4+6$

$=4k^2-4k+3=(2k^2-2)+3$

$=(2k^2-2)$ is even

(b) If x is an even integer then show that x^2+2x+4 is even.

Ans:

Let x be *a* even integer

$x=2k$

Replace x by $2k$ is x^7+2x+4

$x^2+2x+4=(2k)^2+2(2k)+4$

$=4k^2+4k+4$

Q.13 Prove that for any two non-empty sets A and B, $(A\cap B)'=A'\cup B'$

Ans:

L.H.S

$(A\cap B)'$

$x\in(A\cap B)'$

$x\notin A\cap B$

$x\notin A$ and $x\notin B$

$x\in A'$ or $x\in B'$

$x\in A'\cup B'$

$(A\cap B)'\subseteq A'\cup B'$

R.H.S

Let $x\in A'\cup B'$

$x\in A'$ or $x\in B'$

$x\notin A$ and $x\notin B$

$x\notin A\cap B$

$x \in (A \cap B)'$

$A' \cup B' \subseteq (A \cap B)'$

Similarly we can prove that

$(A \cup B)' = A' \cap B'$

Hence Proved L.H.S = R.H.S

Q.14 If x and y are positive real numbers and $x^2 < y^2$ then $x < y$.

Ans:

Let $x = 2$ and $y = 3$

$x < y$

$2 < 3$

$x^2 < y^2$

$(2)^2 < (3)^2$

$4 < 9$

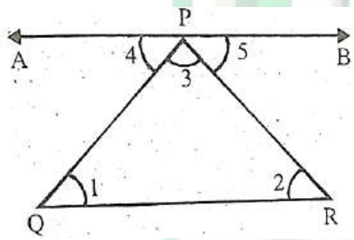
Hence proved $x^2 < y^2$

Q.15 The sum of the interior angles of a triangle is 180°

Ans:

Suppose $\angle A = 40°, \angle B = 70°$,

$\angle C = 70°$



$\angle A + \angle B + \angle C = 180°$

$40° + 70° + 70° = 180°$

$180° = 180°$

If sum of three angle equal to 180°. So we can form triangle

Q.16 If a, b and c are non-zero real numbers, prove that:

(a) $\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$

Ans:

Let $\frac{a}{b} = \frac{c}{d}$

By golden rule of fraction

$\frac{ad}{bd} = \frac{bc}{bd}$

$\left[ad \times \frac{1}{bd}\right]^{bd} = \left[bc \times \frac{1}{bd}\right]^{bd}$

$ad \times \frac{bd}{bd} = bc\left[\frac{bd}{bd}\right]$

$ad = bc$

We shall prove

$ad = bc$

$ad\left[\frac{bd}{bd}\right] = bc\left[\frac{bd}{bd}\right]$

$ad\left[\frac{1}{bd}\right] = bc\left[\frac{1}{bd}\right]$

$\frac{a}{b} = \frac{c}{d}$ golden rule of fraction

(b) $\frac{a}{b}.\frac{c}{d} = \frac{ac}{bd}$

Ans:

L.H.S $= \left[a.\frac{1}{b}\right]\left[\frac{1}{d}.c\right]$

$= a\left[\frac{1}{b}\right]\left[\frac{1}{d}.c\right] = a\left[\left(\frac{1}{b}\right)\left(\frac{1}{d}\right).c\right]$

$= a\left[\frac{1}{b}.\frac{1}{d}.c\right] = a\left[\frac{c}{bd}\right]$

$= \frac{ac}{bd}$

L.H.S = R.H.S

(c) $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

Ans:

L.H.S $\frac{a}{b} + \frac{c}{d}$

$a = \frac{1}{b} + c\frac{1}{d} a \times \frac{1}{b} + c \times \frac{1}{b}$

$= (a+c)\left(\frac{1}{b}\right) = \frac{a+c}{b}$

L.H.S = R.H.S