

Exercise 4.1

QNo: 1 Factorize تجزی کریں

(i) $6x + 12$
 $= 6(x + 2)$

(ii) $15y^2 + 20y$
 $= 5y(3y + 4)$

(iii) $-12x^2 - 3x$
 $= -3x(4x + 1)$

(iv) $4ab + 8ab^2$
 $= 4ab(a + 2b)$

(v) $xy - 3x^2 + 2x$
 $= x(y - 3x + 2)$

(vi) $3a^2b - 9ab^2 + 15ab$
 $= 3ab(a - 3b + 5)$

QNo: 2 Factorize تجزی کریں

(i) $5x + 15$
 $= 5(x + 3)$

(ii) $x^2 + 4x + 3$
 $= x^2 + 3x + x + 3$
 $= x(x + 3) + 1(x + 3)$
 $= (x + 3)(x + 1)$

(iii) $x^2 + 6x + 8$
 $= x^2 + 4x + 2x + 8$
 $= x(x + 4) + 2(x + 4)$
 $= (x + 4)(x + 2)$

(iv) $x^2 + 4x + 4$
 $= x^2 + 2x + 2x + 4$
 $= x(x + 2) + 2(x + 2)$
 $= (x + 2)(x + 2)$
 $= (x + 2)^2$

QNo: 3 Factorize تجزی کریں

(i) $x^2 + x - 12$
 $= x^2 + 4x - 3x - 12$
 $= x(x + 4) - 3(x + 4)$
 $= (x + 4)(x - 3)$

(ii) $x^2 + 7x + 10$
 $= x^2 + 5x + 2x + 10$
 $= x(x + 5) + 2(x + 5)$
 $= (x + 5)(x + 2)$

(iii) $x^2 - 6x + 8$
 $= x^2 - 4x - 2x + 8$
 $= x(x - 4) - 2(x - 4)$
 $= (x - 4)(x - 2)$

(iv) $x^2 - x - 56$
 $= x^2 - 8x + 7x - 56$
 $= x(x - 8) + 7(x - 8)$
 $= (x - 8)(x + 7)$

(v) $x^2 - 10x - 24$
 $= x^2 - 12x + 2x - 24$
 $= x(x - 12) + 2(x - 12)$
 $= (x - 12)(x + 2)$

(vi) $y^2 + 4y - 12$
 $= y^2 + 6y - 2y - 12$
 $= y(y + 6) - 2(y + 6)$
 $= (y + 6)(y - 2)$

(26)

vii) $y^2 + 13y + 36$
 $= y^2 + 9y + 4y + 36$
 $= y(y + 9) + 4(y + 9)$
 $= (y + 9)(y + 4)$

$\begin{array}{r} 36 \times 1 \\ 18 \times 2 \\ 12 \times 3 \\ 9 \times 4 \end{array}$

viii) $x^2 - x - 2$
 $= x^2 - 2x + x - 2$
 $= x(x - 2) + 1(x - 2)$
 $= (x - 2)(x + 1)$

QNo: 4 Factorize تجزی کریں

i) $2x^2 + 7x + 3$
 $= 2x^2 + 6x + x + 3$
 $= 2x(x + 3) + 1(x + 3)$
 $= (x + 3)(2x + 1)$

$\begin{array}{r} 6 \times 1 \\ 3 \times 2 \end{array}$

ii) $2x^2 + 11x + 15$
 $= 2x^2 + 6x + 5x + 15$
 $= 2x(x + 3) + 5(x + 3)$
 $= (x + 3)(2x + 5)$

$\begin{array}{r} 30 \times 1 \\ 15 \times 2 \\ 10 \times 3 \end{array}$

iii) $4x^2 + 13x + 3$
 $= 4x^2 + 12x + x + 3$
 $= 4x(x + 3) + 1(x + 3)$
 $= (x + 3)(4x + 1)$

$\begin{array}{r} 12 \times 1 \\ 4 \times 3 \end{array}$

iv) $3x^2 + 5x + 2$
 $= 3x^2 + 3x + 2x + 2$
 $= 3x(x + 1) + 2(x + 1)$
 $= (x + 1)(3x + 2)$

$\begin{array}{r} 6 \times 1 \\ 3 \times 2 \end{array}$

v) $3y^2 - 11y + 6$
 $= 3y^2 - 9y - 2y + 6$
 $= 3y(y - 3) - 2(y - 3)$
 $= (y - 3)(3y - 2)$

$\begin{array}{r} 18 \times 1 \\ 9 \times 2 \end{array}$

vi) $2y^2 - 5y + 2$
 $= 2y^2 - 4y - y + 2$
 $= 2y(y - 2) - 1(y - 2)$
 $= (y - 2)(2y - 1)$

$\begin{array}{r} 4 \times 1 \\ 2 \times 2 \end{array}$

vii) $4z^2 - 11z + 6$
 $= 4z^2 - 8z - 3z + 6$
 $= 4z(z - 2) - 3(z - 2)$
 $= (z - 2)(4z - 3)$

$\begin{array}{r} 24 \times 1 \\ 12 \times 2 \\ 8 \times 3 \end{array}$

viii) $6 + 7x - 3x^2$
 $= 6 + 9x - 2x - 3x^2$
 $= 3(2 + 3x) - x(2 + 3x)$
 $= (2 + 3x)(3 - x)$

$\begin{array}{r} 18 \times 1 \\ 9 \times 2 \end{array}$

Exercise 4.2

QNo: 1 Factorize تجزی کریں

(i) $4x^4 + 81y^4$
 $= (2x^2)^2 + (9y^2)^2$
 $= (2x^2)^2 + (9y^2)^2 + 2(2x^2)(9y^2) - 2(2x^2)(9y^2)$
 $= (2x^2 + 9y^2)^2 - 36x^2y^2$
 $= (2x^2 + 9y^2)^2 - (6xy)^2$
 $= (2x^2 + 9y^2 + 6xy)(2x^2 + 9y^2 - 6xy)$

$$(ii) a^4 + 64b^4$$

$$= (a^2)^2 + (8b^2)^2$$

$$= (a^2)^2 + (8b^2)^2 + 2(a^2)(8b^2) - 2(a^2)(8b^2)$$

$$= (a^2 + 8b^2)^2 - 16a^2b^2$$

$$= (a^2 + 8b^2)^2 - (4ab)^2$$

$$= (a^2 + 8b^2 + 4ab)(a^2 + 8b^2 - 4ab)$$

$$(iii) x^4 + 4x^2 + 16$$

$$= x^4 + 16 + 4x^2$$

$$= (x^2)^2 + (4)^2 + 2(x^2)(4) - 2(x^2)(4) + 4x^2$$

$$= (x^2 + 4)^2 - 8x^2 + 4x^2$$

$$= (x^2 + 4)^2 - 4x^2$$

$$= (x^2 + 4)^2 - (2x)^2$$

$$= (x^2 + 4 + 2x)(x^2 + 4 - 2x)$$

$$= (x^2 + 2x + 4)(x^2 - 2x + 4)$$

$$(iv) x^4 - 14x^2 + 1$$

$$= x^4 + 1 - 14x^2$$

$$= (x^2)^2 + (1)^2 + 2(x^2)(1) - 2(x^2)(1) - 14x^2$$

$$= (x^2 + 1)^2 - 2x^2 - 14x^2$$

$$= (x^2 + 1)^2 - 16x^2$$

$$= (x^2 + 1)^2 - (4x)^2$$

$$= (x^2 + 1 + 4x)(x^2 + 1 - 4x)$$

$$= (x^2 + 4x + 1)(x^2 - 4x + 1)$$

$$(v) x^4 - 30x^2y^2 + 9y^4$$

$$= x^4 + 9y^4 - 30x^2y^2$$

$$= (x^2)^2 + (3y^2)^2 + 2(x^2)(3y^2) - 2(x^2)(3y^2) - 30x^2y^2$$

$$= (x^2 + 3y^2)^2 - 6x^2y^2 - 30x^2y^2$$

$$= (x^2 + 3y^2)^2 - 36x^2y^2$$

$$= (x^2 + 3y^2)^2 - (6xy)^2$$

$$= (x^2 + 3y^2 + 6xy)(x^2 + 3y^2 - 6xy)$$

$$= (x^2 + 6xy + 3y^2)(x^2 - 6xy + 3y^2)$$

$$(vi) x^4 - 7x^2y^2 + y^4$$

$$= x^4 + y^4 - 7x^2y^2$$

$$= (x^2)^2 + (y^2)^2 + 2(x^2)(y^2) - 2(x^2)(y^2) - 7x^2y^2$$

$$= (x^2 + y^2)^2 - 2x^2y^2 - 7x^2y^2$$

$$= (x^2 + y^2)^2 - 9x^2y^2$$

$$= (x^2 + y^2)^2 - (3xy)^2$$

$$= (x^2 + y^2 + 3xy)(x^2 + y^2 - 3xy)$$

$$= (x^2 + 3xy + y^2)(x^2 - 3xy + y^2)$$

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تجزی کریں

$$(i) (x+1)(x+2)(x+3)(x+4) + 1$$

$$= (x+1)(x+4)(x+2)(x+3) + 1$$

$$= (x^2 + 4x + x + 4)(x^2 + 3x + 2x + 6) + 1$$

$$= (x^2 + 5x + 4)(x^2 + 5x + 6) + 1$$

$$\text{Let } x^2 + 5x = y$$

$$= (y + 4)(y + 6) + 1$$

$$= y^2 + 6y + 4y + 24 + 1$$

$$= y^2 + 10y + 25$$

$$= (y)^2 + 2(y)(5) + (5)^2$$

$$= (y + 5)^2$$

یہی قیمت درج کرنے سے

$$= (x^2 + 5x + 5)^2$$

$$(ii) (x+2)(x-7)(x-4)(x-1) + 17$$

$$= (x^2 - 7x + 2x - 14)(x^2 - x - 4x + 4) + 17$$

$$= (x^2 - 5x - 14)(x^2 - 5x + 4) + 17$$

$$\text{Let } x^2 - 5x = y$$

$$= (y - 14)(y + 4) + 17$$

$$= y^2 + 4y - 14y - 56 + 17$$

$$= y^2 - 10y - 39$$

$$= y^2 - 13y + 3y - 39$$

$$= y(y - 13) + 3(y - 13)$$

$$= (y - 13)(y + 3)$$

یہی قیمت درج کرنے سے

$$= (x^2 - 5x - 13)(x^2 - 5x + 3)$$

$$(iii) (2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1$$

$$\text{Let } 2x^2 + 7x = y$$

$$= (y + 3)(y + 5) + 1$$

$$= y^2 + 5y + 3y + 15 + 1$$

$$= y^2 + 8y + 16 = (y)^2 + 2(y)(4) + (4)^2$$

$$= (y + 4)^2$$

یہی قیمت درج کرنے سے

$$= (2x^2 + 7x + 4)^2$$

$$(iv) (3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3$$

$$\text{Let } 3x^2 + 5x = y$$

$$= (y + 3)(y + 5) - 3$$

$$= y^2 + 5y + 3y + 15 - 3$$

$$= y^2 + 8y + 12$$

$$= y^2 + 6y + 2y + 12$$

$$= y(y + 6) + 2(y + 6)$$

$$= (y + 6)(y + 2)$$

یہی قیمت درج کرنے سے

$$\text{Putting value of } y$$

$$= (3x^2 + 5x + 6)(3x^2 + 5x + 2)$$

$$\begin{aligned}
 (v) & (x+1)(x+2)(x+3)(x+6) - 3x^2 \\
 &= (x+1)(x+6)(x+2)(x+3) - 3x^2 \\
 &= (x^2+6x+x+6)(x^2+3x+2x+6) - 3x^2 \\
 &= (x^2+7x+6)(x^2+5x+6) - 3x^2 \\
 &= (x^2+6+7x)(x^2+6+5x) - 3x^2 \\
 &\text{Let } x^2+6 = y
 \end{aligned}$$

$$\begin{aligned}
 &= (y+7x)(y+5x) - 3x^2 \\
 &= y^2+5xy+7xy+35x^2-3x^2 \\
 &= y^2+12xy+32x^2 \\
 &= y^2+8xy+4xy+32x^2 \\
 &= y(y+8x)+4x(y+8x) \\
 &= (y+8x)(y+4x) \quad \text{یہی قیمت درج کرنے سے} \\
 &= (x^2+6+8x)(x^2+6+4x) \\
 &= (x^2+8x+6)(x^2+4x+6)
 \end{aligned}$$

$$\begin{aligned}
 (vi) & (x+1)(x-1)(x+2)(x-2) - 16x^2 \\
 &= (x+1)(x+2)(x-1)(x-2) - 16x^2 \\
 &= (x^2+2x+x+2)(x^2-2x-x+2) - 16x^2 \\
 &= (x^2+3x+2)(x^2-3x+2) - 16x^2 \\
 &= (x^2+2+3x)(x^2+2-3x) - 16x^2
 \end{aligned}$$

$$\begin{aligned}
 &\text{Let } x^2+2 = y \\
 &= (y+3x)(y-3x) - 16x^2 \\
 &= y^2-3xy+3xy-9x^2-16x^2 \\
 &= y^2-25x^2 = (y)^2 - (5x)^2 \\
 &= (y+5x)(y-5x) \quad \text{یہی قیمت درج کرنے سے} \\
 &= (x^2+2+5x)(x^2+2-5x) \\
 &= (x^2+5x+2)(x^2-5x+2)
 \end{aligned}$$

QNo:3 Factorize تجزی کریں

$$\begin{aligned}
 (i) & 8x^3+12x^2+6x+1 \\
 &\text{Using formula } (a+b)^3 = a^3+3a^2b+3ab^2+b^3 \\
 &= (2x)^3+3(2x)^2(1)+3(2x)(1)^2+(1)^3 \\
 &= (2x+1)^3
 \end{aligned}$$

$$\begin{aligned}
 (ii) & 27a^3+108a^2b+144ab^2+64b^3 \\
 &\text{Using formula } (a+b)^3 = a^3+3a^2b+3ab^2+b^3 \\
 &= (3a)^3+3(3a)^2(4b)+3(3a)(4b)^2+(4b)^3 \\
 &= (3a+4b)^3
 \end{aligned}$$

$$\begin{aligned}
 (iii) & x^3+18x^2y+108xy^2+216y^3 \\
 &\text{Using formula } (a+b)^3 = a^3+3a^2b+3ab^2+b^3 \\
 &= (x)^3+3(x)^2(6y)+3(x)(6y)^2+(6y)^3 \\
 &= (x+6y)^3
 \end{aligned}$$

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$$\begin{aligned}
 (iv) & 8x^3-125y^3+150xy^2-60x^2y \\
 &\text{Using formula } (a-b)^3 = a^3-3a^2b+3ab^2-b^3 \\
 &= 8x^3-60x^2y+150xy^2-125y^3 \\
 &= (2x)^3-3(2x)^2(5y)+3(2x)(5y)^2-(5y)^3 \\
 &= (2x-5y)^3
 \end{aligned}$$

QNo:4 Factorize تجزی کریں

$$\begin{aligned}
 (i) & 125a^3-1 \\
 &\text{Using formula } a^3-b^3 = (a-b)(a^2+ab+b^2) \\
 &= (5a)^3-(1)^3 \\
 &= (5a-1)((5a)^2+(5a)(1)+(1)^2) \\
 &= (5a-1)(25a^2+5a+1)
 \end{aligned}$$

$$\begin{aligned}
 (ii) & 64x^3+125 \\
 &\text{Using formula } a^3+b^3 = (a+b)(a^2-ab+b^2) \\
 &= (4x)^3+(5)^3 \\
 &= (4x+5)((4x)^2-(4x)(5)+(5)^2) \\
 &= (4x+5)(16x^2-20x+25)
 \end{aligned}$$

$$\begin{aligned}
 (iii) & x^6-27 \\
 &\text{Using formula } a^3-b^3 = (a-b)(a^2+ab+b^2) \\
 &= (x^2)^3-(3)^3 \\
 &= (x^2-3)((x^2)^2+(x^2)(3)+(3)^2) \\
 &= (x^2-3)(x^4+3x^2+9)
 \end{aligned}$$

$$\begin{aligned}
 (iv) & 1000a^3+1 \\
 &\text{Using formula } a^3+b^3 = (a+b)(a^2-ab+b^2) \\
 &= (10a)^3+(1)^3 \\
 &= (10a+1)((10a)^2-(10a)(1)+(1)^2) \\
 &= (10a+1)(100a^2-10a+1)
 \end{aligned}$$

$$\begin{aligned}
 (v) & 343x^3+216 \\
 &\text{Using formula } a^3+b^3 = (a+b)(a^2-ab+b^2) \\
 &= (7x)^3+(6)^3 \\
 &= (7x+6)((7x)^2-(7x)(6)+(6)^2) \\
 &= (7x+6)(49x^2-42x+36)
 \end{aligned}$$

$$\begin{aligned}
 (vi) & 27-512y^3 \\
 &\text{Using formula } a^3-b^3 = (a-b)(a^2+ab+b^2) \\
 &= (3)^3-(8y)^3 \\
 &= (3-8y)((3)^2+(3)(8y)+(8y)^2) \\
 &= (3-8y)(9+24y+64y^2)
 \end{aligned}$$

Exercise 4.3

QNo:1 Factorizing تجزی کریں اور پھر HCF معلوم کریں

$$\begin{aligned}
 (i) & 21x^2y, 35xy^2 \\
 & 21x^2y = 7 \cdot 3 \cdot x \cdot x \cdot y \\
 & 35xy^2 = 7 \cdot 5 \cdot x \cdot y \cdot y \\
 & \text{Common Factors} = 7 \cdot x \cdot y \\
 & \text{HCF} = 7xy
 \end{aligned}$$

Ans Q
07/04/25

(ii) $4x^2 - 9y^2, 2x^2 - 3xy$

$$4x^2 - 9y^2 = (2x)^2 - (3y)^2$$

$$= (2x+3y)(2x-3y)$$

$$2x^2 - 3xy = x(2x-3y)$$

مشترک عامل (C.F) = $(2x-3y)$

اعظم عامل (HCF) = $(2x-3y)$

(iii) $x^3 - 1, x^2 + x + 1$

$$x^3 - 1 = (x)^3 - (1)^3 \quad \text{formula}$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$= (x-1)(x^2 + (x)(1) + (1)^2)$$

$$= (x-1)(x^2 + x + 1)$$

$$x^2 + x + 1 = (x^2 + x + 1)$$

مشترک عامل (C.F) = $x^2 + x + 1$

اعظم عامل (HCF) = $x^2 + x + 1$

(iv) $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$

$$a^3 + 2a^2 - 3a = a(a^2 + 2a - 3)$$

$$= a(a^2 + 3a - a - 3)$$

$$= a[a(a+3) - 1(a+3)]$$

$$= a(a+3)(a-1)$$

$$2a^3 + 5a^2 - 3a = a(2a^2 + 5a - 3)$$

$$= a(2a^2 + 6a - a - 3)$$

$$= a[2a(a+3) - 1(a+3)]$$

$$= a(a+3)(2a-1)$$

مشترک عامل (C.F) = $a(a+3)$

اعظم عامل (HCF) = $a(a+3)$

(v) $t^2 - 3t - 4, t^2 + 5t + 4, t^2 - 1$

$$t^2 - 3t - 4 = t^2 - 4t + t - 4$$

$$= t(t-4) + 1(t-4)$$

$$= (t-4)(t+1)$$

$$t^2 + 5t + 4 = t^2 + 4t + t + 4$$

$$= t(t+4) + 1(t+4)$$

$$= (t+4)(t+1)$$

$$t^2 - 1 = (t)^2 - (1)^2$$

$$= (t+1)(t-1)$$

مشترک عامل (C.F) = $(t+1)$

اعظم عامل (HCF) = $(t+1)$

(vi) $x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x$

$$x^2 + 15x + 56 = x^2 + 8x + 7x + 56$$

$$= x(x+8) + 7(x+8)$$

$$= (x+8)(x+7)$$

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$$x^2 + 5x - 24 = x^2 + 8x - 3x - 24$$

$$= x(x+8) - 3(x+8)$$

$$= (x+8)(x-3)$$

$$x^2 + 8x = x(x+8)$$

مشترک عامل (C.F) = $(x+8)$

اعظم عامل (HCF) = $(x+8)$

QNo:2 HCF by division method اعظم عامل (HCF) کے لیے تقسیم کا طریقہ

(i) $27x^3 + 9x^2 - 3x - 10, 3x - 2$

$$3x-2 \overline{) 27x^3 + 9x^2 - 3x - 10} \quad (9x^2 + 9x + 5)$$

$$\underline{+ 27x^3 + 18x^2}$$

$$27x^3 - 3x - 10$$

$$\underline{+ 27x^3 + 18x^2}$$

$$15x^2 - 10$$

$$\underline{+ 15x^2 - 10}$$

اعظم عامل (HCF) = $(3x-2) \times$

(ii) $x^3 - 9x^2 + 23x - 15, x^2 - 4x + 3$

$$x^2-4x+3 \overline{) x^3 - 9x^2 + 23x - 15} \quad (x-5)$$

$$\underline{+ x^3 - 4x^2 + 3x}$$

$$-5x^2 + 20x - 15$$

$$\underline{+ 5x^2 - 20x + 15}$$

اعظم عامل = HCF = $x^2 - 4x + 3$

(iii) $2x^3 + 2x^2 + 2x + 2, 6x^3 + 12x^2 + 6x + 12$

$$2x^3 + 2x^2 + 2x + 2 \overline{) 6x^3 + 12x^2 + 6x + 12} \quad (3)$$

$$\underline{+ 6x^3 + 6x^2 + 6x + 6}$$

$$6x^2 + 6$$

چونکہ دونوں کثیر رقمیوں میں 2 مشترک عامل ہے اور (3) نہیں ہے
3 کو ignore کر دیں گے

$$2x^2 + 2 \overline{) 2x^3 + 2x^2 + 2x + 2} \quad (x+1)$$

$$\underline{+ 2x^3 + 2x^2}$$

$$2x^2 + 2$$

$$\underline{+ 2x^2 + 2}$$

اعظم عامل (HCF) = $2x^2 + 2 = 2(x^2 + 1)$

(iv) $2x^3 - 4x^2 + 16x, x^3 - 4x, 3x^2 - 6x$

$$x^3 - 4x \overline{) 2x^3 - 4x^2 + 16x} \quad (2)$$

$$\underline{+ 2x^3 - 8x^2}$$

$$-4x^2 + 16x$$

$$\underline{+ 4x^2 - 8x}$$

$$-4x^2 + 16x$$

$$x^2 + 2x \overline{) x^3 - 4x} \quad (x-2)$$

$$\underline{+ x^3 + 2x^2}$$

$$-2x^2 - 4x$$

$$\underline{+ 2x^2 + 4x}$$

$$x$$

4 - دونوں رقمیوں میں
CF نہیں ہے لہذا اسے
ignore کر سکتے ہیں

$$x^2 + 2x \sqrt{3x^2 + 6x} \begin{array}{r} 3 \\ + 3x^2 + 6x \\ \hline \end{array}$$

$$\text{HCF} = x^2 + 2x = x(x+2)$$

QNo: 3 ذواصفاف اقل (LCM) بزرگه تجزی factorization.

(i) $2a^2b, 4ab^2, 6ab$

$$2a^2b = 2 \cdot a \cdot a \cdot b$$

$$4ab^2 = 2 \cdot 2 \cdot a \cdot b \cdot b$$

$$6ab = 2 \cdot 3 \cdot a \cdot b$$

$$\text{CF} = 2 \cdot a \cdot b$$

$$\text{N.C.F} = a \cdot 2 \cdot b \cdot 3$$

$$\text{LCM} = \text{CF} \times \text{N.C.F}$$

$$= 2 \cdot a \cdot b \cdot a \cdot 2 \cdot b \cdot 3$$

$$= 12a^2b^2$$

(ii) $x^2 + x, x^3 + x^2$

$$x^2 + x = x(x+1)$$

$$x^3 + x^2 = x^2(x+1) = x \cdot x(x+1)$$

$$\text{CF} = x(x+1)$$

$$\text{N.C.F} = x$$

$$\text{LCM} = \text{CF} \times \text{N.C.F}$$

$$= x(x+1) \cdot x$$

$$= x^2(x+1)$$

(iii) $a^2 - 4a + 4, a^2 - 2a$

$$a^2 - 4a + 4 = a^2 - 2a - 2a + 4$$

$$= a(a-2) - 2(a-2)$$

$$= (a-2)(a-2)$$

$$a^2 - 2a = a(a-2)$$

$$\text{CF} = (a-2)$$

$$\text{N.C.F} = a(a-2)$$

$$\text{LCM} = \text{CF} \times \text{N.C.F}$$

$$= (a-2)a(a-2)$$

$$= a(a-2)^2$$

(iv) $x^4 - 16, x^3 - 4x$

$$x^4 - 16 = (x^2)^2 - (4)^2$$

$$= (x^2 + 4)(x^2 - 4)$$

$$= (x^2 + 4)((x)^2 - (2)^2)$$

$$= (x^2 + 4)(x+2)(x-2)$$

$$x^3 - 4x = x(x^2 - 4)$$

$$= x((x)^2 - (2)^2)$$

$$= x(x+2)(x-2)$$

$$\text{CF} = (x+2)(x-2)$$

$$\text{N.C.F} = (x^2 + 4)(x)$$

30 ذواصفاف اقل (LCM) = C.F x N.C.F

$$= (x+2)(x-2)(x^2+4)x$$

$$= (x^2-4)(x^2+4)x$$

$$= x(x^4-16)$$

(v) $16-4x^2, x^2+x-6, 4-x^2$

$$16-4x^2 = 4(4-x^2) = 4((2)^2 - (x)^2)$$

$$= 4(2+x)(2-x)$$

$$x^2+x-6 = x^2+3x-2x-6$$

$$= x(x+3)-2(x+3)$$

$$= (x+3)(x-2) = -(x+3)(2-x)$$

$$4-x^2 = (2)^2 - (x)^2$$

$$= (2+x)(2-x)$$

$$\text{CF} = (2-x)$$

$$\text{N.C.F} = -4(2+x)(x+3)$$

$$\text{LCM} = \text{CF} \times \text{N.C.F}$$

$$= -(2-x)4(2+x)(x+3)$$

$$= -4(4-x^2)(x+3)$$

QNo: 4 عاظم (HCF) = (y-7)

$$\text{LCM} = y^3 - 10y^2 + 11y + 70$$

$$p(x) = y^2 - 5y - 14, q(x) = ?$$

$$p(x) \cdot q(x) = \text{HL}$$

$$q(x) = \frac{\text{HL}}{p(x)} = \frac{(y-7)(y^3 - 10y^2 + 11y + 70)}{y^2 - 5y - 14}$$

$$y^2 - 5y - 14 \begin{array}{r} \sqrt{y^3 - 10y^2 + 11y + 70} \\ y-5 \\ \hline y^3 - 5y^2 - 14y \\ \hline -5y^2 + 25y + 70 \\ \hline -5y^2 + 25y + 70 \\ \hline \end{array}$$

$$q(x) = (y-7)(y-5)$$

$$= y^2 - 5y - 7y + 35$$

$$= y^2 - 12y + 35$$

QNo: 5 ذواصفاف اقل (LCM) = $36x^3(x+a)(x^3-a^3)$

$$\text{HCF} = x^2(x-a)$$

$$p(x) = 4x^2(x^2-a^2), q(x) = ?$$

$$p(x) \cdot q(x) = \text{HL}$$

$$q(x) = \frac{\text{HL}}{p(x)} = \frac{x^2(x-a)36x^3(x+a)(x^3-a^3)}{4x^2(x^2-a^2)}$$

$$= \frac{9x^2(x-a)36x^3(x+a)(x^3-a^3)}{4x^2(x^2-a^2)}$$

$$= 9x^3(x^3-a^3)$$

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QNo: 6 (HCF) = $(x+a)$
 (LCM) = $12x^2(x+a)(x^2-a^2)$
 (Product of two polynomials)
 $= P(x) \cdot Q(x) = ?$

$P(x) \cdot Q(x) = \text{H.L.}$
 $= (x+a) 12x^2 (x+a) (x^2-a^2)$
 $= 12x^2 (x+a) (x+a) (x+a)(x-a)$
 $= 12x^2 (x+a)^3 (x-a)$

Exercise 4.4

QNo: 1 بذریعہ تجزیہ factorization
 square root

(i) $x^2 - 8x + 16$
 $= (x)^2 - 2(x)(4) + (4)^2$
 $= (x-4)^2$
 $\sqrt{x^2 - 8x + 16} = \sqrt{(x-4)^2}$ Taking square root
 $= \pm (x-4)$

(ii) $9x^2 + 12x + 4$
 $= (3x)^2 + 2(3x)(2) + (2)^2$
 $= (3x+2)^2$
 $\sqrt{9x^2 + 12x + 4} = \sqrt{(3x+2)^2}$ جذر المربع لینے سے
 $= \pm (3x+2)$

(iii) $36a^2 + 84a + 49$
 $= (6a)^2 + 2(6a)(7) + (7)^2$
 $= (6a+7)^2$
 $\sqrt{36a^2 + 84a + 49} = \sqrt{(6a+7)^2}$ جذر المربع لینے سے
 $= \pm (6a+7)$

(iv) $64y^2 - 32y + 4$
 $= 4(16y^2 - 8y + 1)$
 $= 4((4y)^2 - 2(4y)(1) + (1)^2)$
 $= 2^2(4y-1)^2$
 $\sqrt{64y^2 - 32y + 4} = \sqrt{2^2(4y-1)^2}$ جذر المربع لینے سے
 $= \sqrt{2^2} \sqrt{(4y-1)^2}$
 $= \pm 2(4y-1)$

31 (v) $200t^2 - 120t + 18$
 $= 2(100t^2 - 60t + 9)$
 $= 2[(10t)^2 - 2(10t)(3) + (3)^2]$
 $= 2(10t-3)^2$ Taking square root
 $\sqrt{200t^2 - 120t + 18} = \sqrt{2(10t-3)^2}$
 $= \sqrt{2} \sqrt{(10t-3)^2}$
 $= \pm \sqrt{2}(10t-3)$

(vi) $40x^2 + 120x + 90$
 $= 10(4x^2 + 12x + 9)$
 $= 10[(2x)^2 + 2(2x)(3) + (3)^2]$
 $= 10(2x+3)^2$
 $\sqrt{40x^2 + 120x + 90} = \sqrt{10(2x+3)^2}$ جذر المربع لینے سے
 $= \sqrt{10} \sqrt{(2x+3)^2}$
 $= \pm \sqrt{10}(2x+3)$

QNo: 2 (division method) بذریعہ تجزیہ (square root)

(i) $4x^4 - 28x^3 + 37x^2 + 42x + 9$
 $2x^2 - 7x - 3$
 $2x^2 \overline{) 4x^4 - 28x^3 + 37x^2 + 42x + 9}$
 $\underline{+ 4x^4}$
 $4x^2 - 7x$ $\underline{- 28x^3 + 37x^2 + 42x + 9}$
 $\underline{+ 28x^3 - 49x^2}$
 $4x^2 - 14x - 3$ $\underline{- 12x^2 + 42x + 9}$
 $\underline{- 12x^2 + 42x + 9}$
 $\times$
 $\sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9} = \pm (2x^2 - 7x - 3)$

(ii) $121x^4 - 198x^3 - 183x^2 + 216x + 144$
 $11x^2 - 9x - 12$
 $11x^2 \overline{) 121x^4 - 198x^3 - 183x^2 + 216x + 144}$
 $\underline{+ 121x^4}$
 $22x^2 - 9x$ $\underline{- 198x^3 - 183x^2 + 216x + 144}$
 $\underline{+ 198x^3 - 81x^2}$
 $22x^2 - 18x - 12$ $\underline{- 264x^2 + 216x + 144}$
 $\underline{- 264x^2 + 216x + 144}$
 $\times$
 $\sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144} = \pm (11x^2 - 9x - 12)$

$$(iii) \begin{array}{r} x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4 \\ \underline{x^2 - 5xy + y^2} \\ 2x^2 - 5xy \\ \underline{-10x^3y + 27x^2y^2 - 10xy^3 + y^4} \\ 2x^2 - 10xy + y^4 \\ \underline{2x^2y^2 - 10xy^3 + y^4} \\ \pm 2x^2y^2 \pm 10xy^3 \pm y^4 \\ \hline x \end{array}$$

$$\sqrt{x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4} = \pm(x^2 - 5xy + y^2)$$

$$(iv) \begin{array}{r} 4x^4 - 12x^3 + 37x^2 - 42x + 49 \\ \underline{2x^2 - 3x + 7} \\ 4x^2 - 3x \\ \underline{-12x^3 + 37x^2 - 42x + 49} \\ -12x^3 + 9x^2 \\ \underline{28x^2 - 42x + 49} \\ \pm 28x^2 \pm 42x \pm 49 \\ \hline x \end{array}$$

$$\sqrt{4x^4 - 12x^3 + 37x^2 - 42x + 49} = \pm(2x^2 - 3x + 7)$$

QNo:3 $R(x) = -x^2 + 6x - 8$

$$\begin{aligned} &= -x^2 + 4x + 2x - 8 \\ &= -x(x-4) + 2(x-4) \\ &= (x-4)(-x+2) \end{aligned}$$

Investment Profit will be zero when

$$\begin{aligned} x-4 &= 0 \quad \text{or} \quad -x+2=0 \\ x &= 4 \quad \text{or} \quad x=2 \end{aligned}$$

Q No:4 $P(x) = x^3 - 15x^2 + 75x - 125$

$$\begin{aligned} &= (x)^3 - 3(x)^2(5) + 3(x)(5)^2 - (5)^3 \\ &= (x-5)^3 \quad \text{by using formula} \\ &\quad (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \end{aligned}$$

Profit will be zero when

$$\begin{aligned} (x-5)^3 &= 0 \\ x-5 &= 0 \\ x &= 5 \end{aligned}$$

32 QNo:5 $V(x) = 2x^3 - 6x^2 + 4x$

$$\begin{aligned} &= x(2x^2 - 6x + 4) \\ &= x(2x^2 - 4x - 2x + 4) \\ &= x[2x(x-2) - 2(x-2)] \\ &= x(x-2)(2x-2) \end{aligned}$$

Potential energy will be zero when

$$\begin{aligned} x &= 0 \quad \text{or} \quad x-2=0 \quad \text{or} \quad 2x-2=0 \\ x &= 0 \quad \text{or} \quad x=2 \quad \text{or} \quad 2x=2 \\ x &= \frac{2}{2} = 1 \end{aligned}$$

QNo:6 $\gamma(x) = 2x^2 - 8x + 6$

$$\begin{aligned} &= 2x^2 - 2x - 6x + 6 \\ &= 2x(x-1) - 6(x-1) \\ &= (x-1)(2x-6) \end{aligned}$$

Points of zero deflection will be, when

$$\begin{aligned} x-1 &= 0 \quad \text{or} \quad 2x-6=0 \\ x &= 1 \quad \text{or} \quad 2x=6 \\ x &= \frac{6}{2} = 3 \end{aligned}$$

R. Exercise 4

QNo:2 factorize تجزیه کنید

(i) $4x^3 + 18x^2 - 12x = 2x(x^2 + 9x - 6)$

$$= 2x(2x^2 + 9x - 6)$$

(ii) $x^3 + 64y^3$

$$\begin{aligned} &= (x)^3 + (4y)^3 \quad \text{using formula } a^3 + b^3 \\ &= (x+4y)[(x)^2 - (x)(4y) + (4y)^2] = (a+b)(a^2 - ab + b^2) \\ &= (x+4y)(x^2 - 4xy + 16y^2) \end{aligned}$$

(iii) $x^3y^3 - 8$

$$\begin{aligned} &= (xy)^3 - (2)^3 \quad \text{using formula } a^3 - b^3 \\ &= (xy-2)[(xy)^2 + (xy)(2) + (2)^2] = (a-b)(a^2 + ab + b^2) \\ &= (xy-2)(x^2y^2 + 2xy + 4) \end{aligned}$$

(iv) $-x^2 - 23x - 60$

$$\begin{aligned} &= -x^2 - 20x - 3x - 60 \\ &= -x(x+20) - 3(x+20) \\ &= (x+20)(-x-3) \end{aligned}$$

(v) $2x^2 + 7x + 3$

$$\begin{aligned} &= 2x^2 + 6x + x + 3 \\ &= 2x(x+3) + 1(x+3) \\ &= (x+3)(2x+1) \end{aligned}$$

$$(vi) x^4 + 64$$

$$= (x^2)^2 + (8)^2$$

$$= (x^2)^2 + (8)^2 + 2(x^2)(8) - 2(x^2)(8)$$

$$= (x^2 + 8)^2 - 16x^2$$

$$= (x^2 + 8)^2 - (4x)^2$$

$$= (x^2 + 8 + 4x)(x^2 + 8 - 4x)$$

$$(vii) x^4 + 2x^2 + 9$$

$$= x^4 + 9 + 2x^2$$

$$= (x^2)^2 + (3)^2 + 2(x^2)(3) - 2(x^2)(3) + 2x^2$$

$$= (x^2 + 3)^2 - 6x^2 + 2x^2$$

$$= (x^2 + 3)^2 - 4x^2$$

$$= (x^2 + 3)^2 - (2x)^2$$

$$= (x^2 + 3 + 2x)(x^2 + 3 - 2x)$$

$$= (x^2 + 2x + 3)(x^2 - 2x + 3)$$

$$(viii) (x+3)(x+4)(x+5)(x+6) - 360$$

$$= (x+3)(x+6)(x+4)(x+5) - 360$$

$$= (x^2 + 6x + 3x + 18)(x^2 + 5x + 4x + 20) - 360$$

$$= (x^2 + 9x + 18)(x^2 + 9x + 20) - 360$$

$$\text{فرض کیا } x^2 + 9x = y$$

$$= (y + 18)(y + 20) - 360$$

$$= y^2 + 20y + 18y + 360 - 360$$

$$= y^2 + 38y$$

$$= y(y + 38)$$

$$= (x^2 + 9x)(x^2 + 9x + 38)$$

$$= x(x+9)(x^2 + 9x + 38)$$

$$(ix) (x^2 + 6x + 3)(x^2 + 6x - 9) + 36$$

$$\text{فرض کیا } x^2 + 6x = y$$

$$= (y + 3)(y - 9) + 36$$

$$= y^2 - 9y + 3y - 27 + 36$$

$$= y^2 - 6y + 9$$

$$= y^2 - 3y - 3y + 9$$

$$= y(y - 3) - 3(y - 3)$$

$$= (y - 3)(y - 3)$$

$$= (y - 3)^2$$

$$\text{یا کی قیمت درج کرنے سے } y^2 - 6y + 9 = (y - 3)^2$$

$$= (x^2 + 6x - 3)^2$$

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33 QNo: 3 عدا اعظم اور ذواصف اقل معلوم کریں LCM HCF

$$i) 4x^3 + 12x^2, 8x^2 + 16x$$

$$4x^3 + 12x^2 = 4x^2(x + 3) = 2 \cdot 2 \cdot x \cdot x(x + 3)$$

$$8x^2 + 16x = 8x(x + 2) = 2 \cdot 2 \cdot 2 \cdot x(x + 2)$$

$$\text{مشترک عدا (C.F.)} = 2 \cdot 2 \cdot x \Rightarrow \text{عدا اعظم (HCF)} = 4x$$

$$\text{ذواصف اقل (LCM)} = C.F. \times N.C.F. = x(x+3) \cdot 2(x+2)$$

$$= 2 \cdot 2 \cdot x \cdot x(x+3) \cdot 2(x+2)$$

$$= 8x^2(x+3)(x+2)$$

$$\text{رہے ہیں}$$

$$ii) x^3 + 3x^2 - 4x, x^2 - 4x + 3$$

$$x^3 + 3x^2 - 4x = x(x^2 + 3x - 4)$$

$$= x(x^2 + 4x - x - 4) = x(x(x+4) - 1(x+4))$$

$$= x(x+4)(x-1)$$

$$x^2 - 4x + 3 = x^2 - 3x - x + 3 = x(x-3) - 1(x-3)$$

$$= (x-3)(x-1)$$

$$\text{مشترک عدا (C.F.)} = (x-1) \Rightarrow \text{عدا اعظم (HCF)} = x-1$$

$$\text{ذواصف اقل (LCM)} = C.F. \times N.C.F. = (x-1)x(x+4)(x-3)$$

$$iii) x^2 + 8x + 16, x^2 - 16$$

$$x^2 + 8x + 16 = (x)^2 + 2(x)(4) + (4)^2 = (x+4)^2$$

$$x^2 - 16 = (x)^2 - (4)^2 = (x+4)(x-4)$$

$$\text{مشترک عدا (C.F.)} = (x+4) \Rightarrow \text{عدا اعظم} = (x+4)$$

$$\text{ذواصف اقل (LCM)} = C.F. \times N.C.F. = (x+4)(x+4)(x-4)$$

$$= (x+4)^2(x-4)$$

$$iv) x^3 - 9x, x^2 - x - 6$$

$$x^3 - 9x = x(x^2 - 9) = x(x^2 - (3)^2) = x(x+3)(x-3)$$

$$x^2 - x - 6 = x^2 - 3x + 2x - 6 = x(x-3) + 2(x-3) = (x-3)(x+2)$$

$$\text{مشترک عدا (C.F.)} = (x-3) \Rightarrow \text{عدا اعظم (HCF)} = (x-3)$$

$$\text{ذواصف اقل (LCM)} = C.F. \times N.C.F. = (x-3)x(x+3)(x+2)$$

$$= x(x^2 - 9)(x+2)$$

$$\text{جزء المرجع بزرگترین اور بزرگترین تقسیم معلوم کریں}$$

$$16x^4 + 8x^2 + 1 = (4x^2)^2 + 2(4x^2)(1) + (1)^2$$

$$= (4x^2 + 1)^2$$

$$\sqrt{16x^4 + 8x^2 + 1} = \sqrt{(4x^2 + 1)^2} = \pm(4x^2 + 1)$$

$$QNo: 5 C(x) = x^2 - 8x + 15$$

$$= x^2 - 5x - 3x + 15$$

$$= x(x-5) - 3(x-5)$$

$$= (x-5)(x-3)$$

Optimal repayment period for

Hurria's loan will be, when

$$x-5=0 \text{ or } x-3=0$$

$$x=5 \text{ years or } x=3 \text{ years}$$