# Exercise # 1.1

**Q#1** Find the multiplicative inverse of complex Numbers.

(i) $(-4, 7)$

$Z = -4 + 7i$

$2 \times \frac{1}{2} = 1$

$$\frac{1}{Z} = \frac{1}{-4+7i} \times \frac{-4-7i}{-4-7i}$$

$$= \frac{-4-7i}{(-4)^2 - (7i)^2}$$

$$\frac{-4-7i}{16-49i^2} \Rightarrow \frac{-4-7i}{16+49}$$

$$\frac{1}{Z} = \frac{-4-7i}{65} \Rightarrow \left(\frac{-4}{65}, \frac{-7}{65}\right)$$

(ii) $(\sqrt{2}, -\sqrt{5})$

$Z = \sqrt{2} - \sqrt{5}\,i$

$$\frac{1}{Z} = \frac{1}{\sqrt{2}-\sqrt{5}\,i} \times \frac{\sqrt{2}+\sqrt{5}\,i}{\sqrt{2}+\sqrt{5}\,i}$$

$$= \frac{\sqrt{2}+\sqrt{5}\,i}{(\sqrt{2})^2 - (\sqrt{5}\,i)^2} \Rightarrow \frac{\sqrt{2}+\sqrt{5}\,i}{2-5i^2}$$

$$\frac{\sqrt{2}+\sqrt{5}\,i}{2+5} \Rightarrow \frac{\sqrt{2}+\sqrt{5}\,i}{7} \Rightarrow \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$$

(iii) $(1, 0)$

$Z = 1 + 0i$

$$\frac{1}{Z} = \frac{1}{1+0i} \times \frac{1-0i}{1-0i} = (1, 0)$$

$x + iy$

## Q#2:- Separate into Real and Imaginary parts

(i) $\dfrac{2-7i}{4+5i}$

$$\frac{2-7i}{4+5i} \times \frac{4-5i}{4-5i}$$

$$\frac{(2-7i)(4-5i)}{(4)^2-(5i)^2}$$

$$= \frac{8-10i-28i+35i^2}{16-25i^2}$$

$$\frac{8-38i+35(-1)}{16-25(-1)}$$

$$\frac{8-38i-35}{16+25} \Rightarrow \frac{-27-38i}{41}$$

$$= -27/41, \frac{-38}{41}$$

(ii) $\dfrac{(-2+3i)^2}{1+i}$

$= \dfrac{4+9i^2-12i}{1+i}$

$= \dfrac{4+9(-1)-12i}{1+i}$

$= \dfrac{4-9-12i}{1+i}$

$= \dfrac{-5-12i}{1+i}$

$\dfrac{-5-12i}{1+i} \times \dfrac{1-i}{1-i}$

$= \dfrac{(-5-12i)(1-i)}{(1)^2-(i)^2}$

$= \dfrac{-5+5i-12i+12i^2}{1-i^2}$

$\dfrac{-5-7i+12(-1)}{1-(-1)} \Rightarrow \dfrac{-5-7i-12}{1+1}$

$= \dfrac{-17-7i}{2} \Rightarrow -\dfrac{17}{2}, \dfrac{-7i}{2}$

Real part $= \dfrac{-17}{2}$

Imaginary part $= -7/2$

(iii) $\frac{i}{1+i}$

$\frac{i}{1+i} \times \frac{1-i}{1-i}$

$\frac{i(1-i)}{(1)^2-(i)^2} \Rightarrow \frac{i-i^2}{1-i^2}$

$= \frac{i-(-1)}{1-(-1)} \Rightarrow \frac{i+1}{1+1}$

$\frac{1+i}{2} \Rightarrow \left(\frac{1}{2}, \frac{i}{2}\right)$

Real part $= 1/2$
Img part $= 1/2$

(iv) $\frac{(4+3i)^2}{4-3i}$

$= \frac{16+9i^2+24i}{4-3i}$

$= \frac{16+9(-1)+24i}{4-3i} \Rightarrow \frac{16-9+24i}{4-3i}$

$\frac{7+24i}{4-3i} \times \frac{4+3i}{4+3i}$

$\frac{(7+24i)(4+3i)}{(4)^2-(3i)^2}$

$\frac{28+21i+96i+72i^2}{16-9i^2}$

$\frac{117i-44}{16+9} = \left(\frac{-44}{25}, \frac{117}{25}\right)$

## Q#3 Prove that $\bar{Z} = Z$ iff $Z$ is real.

Let $Z = a + ib \Rightarrow \bar{Z} = a - ib$

Suppose $\bar{Z} = Z$ then we will prove that $Z$ is real

As $\bar{Z} = Z$

$a - ib = a + ib$

$a - ib - a - ib = 0$

$-2ib = 0$

$-2 \neq 0$, $i \neq 0$

$b = 0$ This shows That $Z$ is real.

## Q#4 For $Z \in \mathbb{C}$, show that

(i) $\dfrac{Z + \bar{Z}}{2} = Re(Z)$

Let $Z = a + bi \Rightarrow \bar{Z} = a - bi$

$$\frac{Z + \bar{Z}}{2} = \frac{a + bi + a - bi}{2}$$

$$= \frac{2a}{2} = a$$

$$\frac{Z + \bar{Z}}{2} = Re(Z)$$

— proved —

(ii) $\frac{Z-\bar{Z}}{2i} = Im(Z)$

Let $Z = a+bi \Rightarrow \bar{Z} = a-bi$

$\frac{Z-\bar{Z}}{2i} = \frac{(a+bi)-(a-bi)}{2i}$

$= \frac{a+bi-a+bi}{2i} \Rightarrow \frac{2bi}{2i}$

$\frac{Z-\bar{Z}}{2i} = b$

$\frac{Z-\bar{Z}}{2i} = Im(Z)$ proved

(iii) $|Z|^2 = Z\cdot\bar{Z}$

Let $Z = a+bi \Rightarrow \bar{Z} = a-bi$

L.H.S $|Z|^2$

$|Z| = \sqrt{a^2+b^2}$

$|Z|^2 = \left(\sqrt{a^2+b^2}\right)^2 \Rightarrow |Z|^2 = a^2+b^2$

R.H.S $Z\cdot\bar{Z}$

$= (a+bi)\cdot(a-bi)$

$(a)^2-(bi)^2 \Rightarrow a^2-b^2i^2$

$a^2-b^2(-1) \Rightarrow a^2+b^2$

L.H.S = R.H.S

**Q#5:** If $Z_1 = 2+i$, $Z_2 = 3-2i$, $Z_3 = 1+3i$ then express $\dfrac{\bar{Z}_1\bar{Z}_3}{Z_2}$ in form of $a+ib$.

$\bar{Z}_1 = 2-i$ , $\bar{Z}_3 = 1-3i$

$$\frac{\bar{Z}_1\bar{Z}_3}{Z_2} = \frac{(2-i)(1-3i)}{(3-2i)}$$

$$= \frac{2-6i-i+3i^2}{(3-2i)}$$

$$= \frac{2-7i+3(-1)}{(3-2i)} \Rightarrow \frac{-1-7i}{3-2i}$$

$$\frac{-1-7i}{3-2i} \times \frac{3+2i}{3+2i}$$

$$= \frac{(-1-7i)(3+2i)}{(3)^2-(2i)^2}$$

$$= \frac{-3-2i-21i-14i^2}{9-4i^2}$$

$$= \frac{-3-23i-14(-1)}{9-4(-1)}$$

$$\frac{-3-23i+14}{9+4} \Rightarrow \frac{11-23i}{13}$$

$$\left(\frac{11}{13} - \frac{23}{13}i\right) \text{ Ans}$$

**Q#6** if $Z_1 = 2+7i$, $Z_2 = -5+3i$ then evaluate

(i) $|2Z_1 - 4Z_2|$

$2Z_1 = 2(2+7i) \Rightarrow 4+14i$

$4Z_2 = 4(-5+3i) = -20+12i$

$2Z_1 - 4Z_2 = (4+14i) - (-20+12i)$

$= 4+14i+20-12i$

$= 24+2i$

$|2Z_1 - 4Z_2| = \sqrt{(24)^2 + (2)^2}$

$= \sqrt{576+4}$

$= \sqrt{580} \Rightarrow 24.08$

OR $= 2\sqrt{145}$

(ii) $|3Z_1 + 2\bar{Z}_1|$

$3Z_1 = 3(2+7i) = 6+21i$

$\bar{Z}_1 = 2-7i$

$2\bar{Z}_1 = 2(2-7i) = 4-14i$

$3Z_1 + 2\bar{Z}_1 = 6+21i+4-14i$

$3Z_1 + 2\bar{Z}_1 = 10+7i$

$|3Z_1 + 2\bar{Z}_1| = \sqrt{(10)^2 + (7)^2}$

$= \sqrt{100+49}$

$= \sqrt{149}$ Ans

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(iii) $\left|-7Z_2 + 2\bar{Z}_2\right|$

$-7Z_2 = -7(-5+3i) = 35-21i$

$2\bar{Z}_2 = 2(-5-3i) = -10-6i$

$-7Z_2 + 2\bar{Z}_2 = 35-21i-10-6i$

$\left|-7Z_2 + 2\bar{Z}_2\right| = \sqrt{(25)^2 + (-27)^2}$

$\left|-7Z_2 + 2\bar{Z}_2\right| = \sqrt{625 + 729}$

$\left|-7Z_2 + 2\bar{Z}_2\right| = \sqrt{1354}$

Ans

(iv) $\left|(Z_1 + Z_2)^3\right|$

$Z_1 + Z_2 = (2+7i) + (-5+3i)$

$Z_1 + Z_2 = -3+10i$

$(Z_1 + Z_2)^3 = (-3+10i)^3$

$= (3)^3 - 3(9)(10i) + 3(3)(100i^2) - (10i)^3$

$= 27 - 270i + 900i^2 - 1000i^3$

$= 27 - 270i - 900 + 1000i$

$= -873 + 973i$

$\left|(Z_1+Z_2)^3\right| = \sqrt{(-873)^2 + (973)^2}$

$= \sqrt{762129 + 946729}$

$\sqrt{1708858} \Rightarrow 109\sqrt{109}$ Ans

**Q #7:** Show that: $i^{n+1} + i^{n+2} + i^{n+3} + i^{n+4} = 0$
for all $n \in N$

We know that

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = i \times i^2 = -i$

$i^4 = (i^2)^2 = 1$

$= i^{n+1} + i^{n+2} + i^{n+3} + i^{n+4}$

$= i^{n+1} + i^{n+1+1} + i^{n+1+2} + i^{n+1+3}$

$= i^{n+1} + i^{n+1} \cdot i + i^{n+1} \cdot i^2 + i^{n+1} \cdot i^3$

$= i^{n+1} \left[ 1 + i + i^2 + i^3 \right]$

$i^{n+1} \left[ 1 + \sqrt{-1} - 1 - i \right] \Rightarrow i^{n+1} \left[ 1 + \sqrt{-1} - 1 - \sqrt{-1} \right]$

$i^{n+1} (0) = 0$

**Q#8** Find the least positive value of $n$, if $\left(\frac{1+i}{1-i}\right)^{2n} = 1$

$$\frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$\frac{(1+i)^2}{(1)^2-(i)^2} \Rightarrow \frac{1+i^2+2i}{1-i^2}$$

$$\frac{1-1+2i}{1+1} \Rightarrow \frac{2i}{2}$$

$$\frac{1+i}{1-i} = i$$

$$(i)^{2n} = 1$$

$$(i)^{2n} = (i)^4$$

$$2n = 4 \Rightarrow n = 4/2 = 2$$

**Q#9** Show that the value of $i^n$ for $n \in N$ and $n > 4$ is $i^r$ where $r$ is the remainder when $n$ is divided by 4.

$$4\overline{)11}\;2,\; 8,\; 3 \qquad 4\overline{)13}\;3,\; 12,\; 1$$

Sol:-

We have to show that $i^n = i^r$

$N$ is a natural number divided by 4 and remainder is $r$. So the relation is expressed as

$$n = 4q + r$$

$$LHS = i^n$$

$$(i)^{4q+r}$$

$= (i)^{4q+r}$

$= (i)^{4q} * (i)^{r}$

$= [(i)^4]^q \times i^r \Rightarrow [(i^2)^2]^q \times i^r$

$= [(-1)^2]^q \times i^r$

$= (1)^q \times i^r$

$= 1 \times i^r = i^r$

$= \cancel{R}$

LHS = RHS