EXERCISE # 1.2 Unit # 1 (Complex Numbers)

Q # 1 Find the values of x and y

(i) $x + iy + 2 - 3i = i(5-i)(3+4i)$

Sol:- $x + iy + 2 - 3i = (5i - i^2)(3+4i)$

$(x+2) + (y-3)i = (5i+1)(3+4i)$

$(x+2) + (y-3)i = 15i + 20i^2 + 3 + 4i$

$(x+2) + (y-3)i = 19i - 20 + 3$

$(x+2) + (y-3)i = 19i - 17$

Comparing Real and Imaginary parts

$x + 2 = -17$

$x = -17 - 2$

$\boxed{x = -19}$

$y - 3 = 19$

$y = 19 + 3$

$\boxed{y = 22}$

(ii) $(x+iy)(1-i) = (2-3i)(-5+5i)\left(\frac{-3i}{5}\right)$

Sol:- $x - xi + iy - yi^2 = 5(2-3i)(-1+i)\left(\frac{-3i}{5}\right)$

$x - xi + iy + y = (2-3i)(-1+i)(-3i)$

$(x+y)+(y-x)i = (2-3i)(3i-3i^2)$

$(x+y)+(y-x)i = (2-3i)(3i+3)$

$(x+y)+(y-x)i = 6i+6-9i^2-9i$

$(x+y)+(y-x)i = -3i+6+9$

$(x+y)+(y-x)i = -3i+15$

Comparing Real and Imaginary Parts

$x+y = 15$ — (i) $\qquad y-x = -3$ — (ii)

$y = \cancel{-3+x}$

Adding Eq (i) and (ii)

$x+y = 15$

$-x+y = -3$

$2y = 12$

$\boxed{y=6}$

put $y=6$ in (ii)

$6-x = -3$

$6+3 = x$

$9 = x$

$\boxed{x=9}$

(iii) $\dfrac{x}{2+i} + \dfrac{y}{3-i} = 4+5i$

$\dfrac{x(3-i)+y(2+i)}{(2+i)(3-i)} = 4+5i$

$\dfrac{3x-xi+2y+yi}{6-2i+3i-i^2} = 4+5i$

$\dfrac{3x+2y+yi-xi}{6+i+1} = 4+5i$

$(3x+2y)+(y-x)i = (4+5i)(7+i)$

$(3x+2y)+(y-x)i = 28+4i+35i+5i^2$

$(3x+2y)+(y-x)i = 28+39i-5$

$(3x+2y)+(y-x)i = 23+39i$

Comparing real and Img parts

$3x+2y = 23$ — (i) $\quad y-x = 39$ — (ii)

Multiplying Eq (ii) by 3 and adding

$-\cancel{3x}+3y = 117$

$\underline{\cancel{3x}+2y = 23}$

$5y = 140$

$y = \dfrac{140}{5} \Rightarrow \boxed{y = 28}$

put $y = 28$ in (ii)

$28 - x = 39$

$28 - 39 = x$

$\boxed{x = -11}$

Q#2 If $Z_1 = -13 + 24i$, $Z_2 = x + yi$, find the values of such that $Z_1 - Z_2 = -27 + 15i$

Given $Z_1 - Z_2 = -27 + 15i$

$(-13 + 24i) - (x + yi) = -27 + 15i$

$-13 + 24i - x - yi = -27 + 15i$

$(-13 - x) + (24 - y)i = -27 + 15i$

Comparing Real and Img parts.

$-13 - x = -27$	$24 - y = 15$
$-13 + 27 = x$	$24 - 15 = y$
$\boxed{x = 14}$	$\boxed{y = 9}$

Exercise # 1.2 (Complex Numbers)

Q#3 Find the value of x and y if

(i) $(x+iy)^2 = 25+60i$

Taking square root on both sides.

$$\sqrt{(x+iy)^2} = \pm\sqrt{25+60i}$$

$$(x+iy) = \pm\sqrt{25+60i}$$

First we solve the part $\sqrt{25+60i}$

Let $Z = 25+60i$ $\quad x=25 \quad y=60$

$$|Z| = \sqrt{(25)^2+(60)^2}$$

$$= \sqrt{625+3600}$$

$$|Z| = \sqrt{4225} = 65$$

$\cancel{\sqrt{Z}} \neq \cancel{\sqrt{B}}$

Square root Formula

$$\sqrt{Z} = \pm\left[\sqrt{\frac{|Z|+x}{2}} + \sqrt{\frac{|Z|-x}{2}}\,\frac{yi}{|y|}\right]$$

$$= \pm\left[\sqrt{\frac{65+25}{2}} + \sqrt{\frac{65-25}{2}}\,\frac{60i}{|60|}\right]$$

$$= \pm\left[\sqrt{\frac{90}{2}} + \sqrt{\frac{40}{2}}\left(\frac{60i}{60}\right)\right]$$

$$\sqrt{Z} = \pm\left[\sqrt{45} + \sqrt{20}\,i\right]$$

$$\sqrt{Z} = \pm\left[\sqrt{45} + \sqrt{20}\,i\right]$$

$$\sqrt{Z} = \pm\left(3\sqrt{5} + 2\sqrt{5}i\right)$$

$$x = \pm 3\sqrt{5}, \quad y = \pm 2\sqrt{5}i$$

Q#3(ii) $(x+iy)^2 = 64+48i$

Taking square root on Both sides

$$\sqrt{(x+iy)^2} = \pm\sqrt{64+48i}$$

$$x+iy = \pm\sqrt{64+48i} \quad \text{—(i)}$$

Let $Z = 64+48i$ and

$$\sqrt{Z} = \sqrt{64+48i}$$

Now $\sqrt{Z} = \sqrt{\frac{|Z|+x}{2}} + \sqrt{\frac{|Z|-x}{2}}\,\frac{yi}{|y|}$

$|Z| = \sqrt{(64)^2+(48)^2}$

$$|Z| = \sqrt{4096+2304}$$

$$= \sqrt{6400}$$

$$|Z| = 80$$

putting the values x, y and $|Z|$ in the Formula we have

$$\sqrt{Z} = \pm\left[\sqrt{\frac{80+64}{2}} + \sqrt{\frac{80-64}{2}}\left(\frac{48i}{|48|}\right)\right]$$

$$= \pm\left(\sqrt{\frac{144}{2}} + \sqrt{\frac{16}{2}}\,i\right)$$

$$\sqrt{Z} = \sqrt{72} + \sqrt{8}\,i$$

$$\sqrt{Z} = \pm(6\sqrt{2} + 2\sqrt{2}\,i)$$

putting the value of $\sqrt{Z}$ in (i)

$$x+iy = \pm(6\sqrt{2} + 2\sqrt{2}\,i)$$

$x = \pm 6\sqrt{2}$ and $y = \pm 2\sqrt{2}i$

Q#3(iii) $x+iy = \dfrac{-2-5i}{(1+3i)^3}$

Solving $(1+3i)^3$

$= (1)^3 + 3(1)^2(3i) + 3(1)(3i)^2 + (3i)^3$

$= 1 + 9i + 3(9i^2) + (27i^3)$

$= 1 + 9i + 27i^2 + (27i)(i^2)$

$= 1 + 9i - 27 - 27i$

$= 1 - 27 + 9i - 27i$

$= -26 - 18i$

$x+iy = \dfrac{-2-5i}{-26-18i}$

$x+iy = \dfrac{-(2+5i)}{-(26+18i)}$

$x+iy = \dfrac{2+5i}{26+18i} \times \dfrac{26-18i}{26-18i}$

$x+iy = \dfrac{52-36i+130i-90i^2}{(26)^2-(18i)^2}$

$x+iy = \dfrac{52+94i-90(-1)}{676-324i^2}$

$x+iy = \dfrac{52+94i+90}{676+324}$

$= \dfrac{142+94i}{1000}$

$= \dfrac{142}{1000} + \dfrac{94}{1000}i$

$x+iy = \dfrac{71}{500} + \dfrac{47}{500}i$

$x = \dfrac{71}{500}$ $\quad y = \dfrac{47}{500}$

Q#4

If $Z_1 = 2+3i$ and $Z_2 = 1-\alpha$ Find the value of α such that $Im(Z_1 Z_2) = 7$

Sol:- $(Z_1 Z_2) = 7$

$= (2+3i)(1-\alpha) = 7$

$= 2 - 2\alpha + 3i - 3\alpha i = 7$

$\cancel{(Z_1 Z_2)} = (2-2\alpha) + 3(1-\alpha)i = 7$

$\cancel{Im}\ 3(1-\alpha) = 7$

$3 - 3\alpha = 7$

$3 - 7 = 3\alpha$

$3\alpha = -4$

$\boxed{\alpha = -4/3}$

Q#5 $Z_1 = x+yi$ and $Z_2 = a+ib$ Find x and y, a and b such that $Z_1 + Z_2 = 10+4i$ and $Z_1 - Z_2 = 6+2i$

$Z_1 + Z_2 = 10+4i$

$\cancel{x + yi + x + yi}$

$x + yi + a + ib = 10 + 4i$

EX#1.2

Q#5 (Remaining)

$(x+a)+(y+b)i = 10+4i$

Comparing Real and Img parts.

$x+a = 10 \longrightarrow (i)$ $\quad y+b = 4 \longrightarrow (ii)$

Now $Z_1 - Z_2 = 6+2i$ (Given)

$Z_1 - Z_2 = x+yi-a-ib = 6+2i$

$(x-a)+(y-b)i = 6+2i$

$(x-a) = 6 \longrightarrow (iii)$ $\quad y-b=2 \longrightarrow (iv)$

Adding (i) and (iii)

$x+a = 10$

$x-a = 6$

$2x = 16$

$\boxed{x=8}$

put $x=8$ in (i)

$8+a=10$

$a=10-8$

$\boxed{a=2}$

Adding Eq (ii) and (iv)

$y+b=4$

$y-b=2$

$2y = 6$

$\boxed{y=3}$

put $y=3$ in (ii)

$3+b=4$

$b=4-3$

$\boxed{b=1}$

Q#6 Show that $\forall z_1, z_2 \in C$, $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

Let $z_1 = a+bi$ and $z_2 = c+di$ then

$\overline{z_1 z_2} = \overline{(a+bi)\cdot(c+di)}$

$= \overline{ac + adi + bci + bdi^2}$

$= \overline{ac + adi + bci - bd}$

$= \overline{(ac-bd) + (ad+bc)i} \Rightarrow (ac-bd) - (ad+bc)i$

R.H.S $\bar{z}_1 \bar{z}_2$

$\bar{z}_1 \bar{z}_2 = (a-bi)(c-di)$

$= ac - adi - bci + bdi^2$

$= ac - adi - bci - bd$

$(ac-bd) - (ad+bc)i$

proved that L.H.S = R.H.S

Q#7 Find the square root of the following Complex Numbers.

(i) $-7-24i$

Let $z = -7-24i$ and $|z| = \sqrt{(-7)^2 + (-24)^2} \Rightarrow \sqrt{49+576}$

$|z| = \sqrt{625} \Rightarrow 25$

Now $\sqrt{z} = \sqrt{-7-24i}$

Using formula $\sqrt{Z} = \pm\left[\sqrt{\frac{|Z|+x}{2}} + \sqrt{\frac{|Z|-x}{2}}\,\frac{yi}{|y|}\right]$

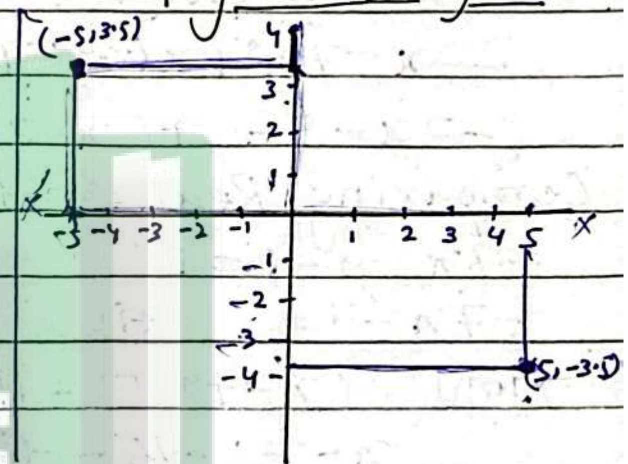
$\sqrt{Z} = \pm\left\{\sqrt{\frac{25-7}{2}} + \sqrt{\frac{25+7}{2}}\,\frac{-24i}{|-24|}\right]$

$= \pm\left[\sqrt{\frac{18}{2}} + \sqrt{\frac{32}{2}}\left(\frac{-24i}{24}\right)\right]$

$\pm\left[\sqrt{9} + \sqrt{16}(-i)\right]$

$\sqrt{Z} = \pm(3-4i)$

Argand Diagram



Q#8 Find the Square Root of $13-20\sqrt{3}i$ and represent them on an Argand diagram.

Let $Z = 13-20\sqrt{3}i$

$|Z| = \sqrt{(13)^2+(-20\sqrt{3})^2}$

$\sqrt{169+1200} \Rightarrow \sqrt{1369} \Rightarrow 37$

$\sqrt{Z} = \pm\left[\sqrt{\frac{37+13}{2}} + \sqrt{\frac{37-13}{2}}\left(\frac{-20\sqrt{3}i}{|-20\sqrt{3}|}\right)\right] \Rightarrow \left[\sqrt{25} + \sqrt{12}\left(\frac{-20\sqrt{3}i}{20\sqrt{3}}\right)\right]$

$\sqrt{Z} = \pm\left[5 + 2\sqrt{3}(-i)\right] \Rightarrow \sqrt{Z} = \pm(5-2\sqrt{3}) \Rightarrow \pm(5, 3.5)$

$(5,-3.5)\ (-5, 3.5)$

Q#10 Find the value of x and y if

$$(5-2i)(x+iy)+3 = i(11-i)-4i$$

$$5x+5yi-2xi-2yi^2+3 = 11i-i^2-4i$$

$$5x+5yi-2xi+2y+3 = 11i+1-4i$$

$$(5x+2y+3)+(5y-2x)i = 1+7i$$

Comparing Real and Img parts

$$5x+2y+3 = 1 \quad ; \quad 5y-2x = 7 \text{ --- (ii)}$$

$$5x+2y = -2 \text{ --- (i)}$$

Multiplying Eq(i) by 2 and Eq(ii) by 5 then adding

$$10x+4y = -4$$

$$-10x+25y = 35$$

$$29y = 31$$

$$\boxed{y = \frac{31}{29}}$$

put $y = \frac{31}{29}$ in Eq(i)

$$5x+2\left(\frac{31}{29}\right) = -2$$

$$5x+\frac{62}{29} = -2$$

$$5x = -\frac{2}{1}-\frac{62}{29}$$

$$5x = \frac{-58-62}{29}$$

$$5x = \frac{-120}{29}$$

$$x = \frac{-120}{29}\times\frac{1}{5} \Rightarrow \boxed{x = \frac{-24}{29}}$$

Q#11 Find the values of u and v if

(i) $(u+iv)^2 = 20+21i$

Square root on Both sides.

$$\sqrt{(u+iv)^2} = \pm\sqrt{20+21i}$$

$$u+iv = \pm\sqrt{20+21i}$$

Let $z = 20+21i$

$$\sqrt{z} = \pm\left[\sqrt{\frac{|z|+x}{2}} + \sqrt{\frac{|z|-x}{2}}\left(\frac{yi}{|y|}\right)\right]$$

$$|z| = \sqrt{(20)^2+(21)^2}$$

$$= \sqrt{400+441}$$

$$= \sqrt{841} \Rightarrow 29$$

$$\sqrt{z} = \pm\left[\sqrt{\frac{29+20}{2}} + \sqrt{\frac{29-20}{2}}\left(\frac{21i}{|21|}\right)\right]$$

$$\sqrt{z} = \pm\left(\sqrt{\frac{49}{2}} + \sqrt{\frac{9}{2}}\, i\right)$$

$$\sqrt{z} = \pm\left(\frac{7}{\sqrt{2}} + \frac{3}{\sqrt{2}} i\right)$$

$$u+iv = \pm\left(\frac{7}{\sqrt{2}} + \frac{3}{\sqrt{2}} i\right)$$

$u = \pm\frac{7}{\sqrt{2}}$ and

$v = \pm 3/\sqrt{2}$

#11 (ii) $(u+iv)^2 = 48-10i$

Taking square root on Both sides.

$$\sqrt{(u+iv)^2} = \pm\sqrt{48-10i}$$

Let $Z = 48-10i$

$$|Z| = \sqrt{(48)^2+(-10)^2}$$

$$= \sqrt{2304+100}$$

$$|Z| = \sqrt{2404} \Rightarrow 2\sqrt{601}$$

$$\sqrt{Z} = \pm\sqrt{\frac{2\sqrt{601}+48}{2}} + \sqrt{\frac{2\sqrt{601}-48}{2}}\left(\frac{-10i}{|-10|}\right)$$

$$= \pm\left[\sqrt{\frac{2(\sqrt{601}+24)}{2}} + \sqrt{\frac{2(\sqrt{601}-24)}{2}}(-i)\right]$$

$$= \pm\left(\sqrt{\sqrt{601}+24} - \sqrt{\sqrt{601}-24}\,i\right)$$

$$u = \pm\sqrt{\sqrt{601}+24} \qquad v = \pm\sqrt{-\sqrt{601}-24}\,i$$

Q#12 If $Z_1 = 4+5i$, $Z_2 = \alpha - 2i$ Find the value of α such that $Re(Z_1Z_2) = 20$

$$(Z_1Z_2) = (4+5i)(\alpha-2i)$$
$$= 4\alpha - 8i + 5\alpha i - 10i^2$$
$$= 4\alpha - 8i + 5\alpha i + 10$$
$$(Z_1Z_2) = (4\alpha+10) + (5\alpha-8)i$$

$$Re(Z_1Z_2) = 4\alpha + 10$$

Given $Re(Z_1Z_2) = 20$

$$4\alpha + 10 = 20$$
$$4\alpha = 20 - 10$$
$$4\alpha = 10$$
$$\alpha = \frac{10}{4}$$
$$\boxed{\alpha = 5/2}$$