EXERCISE # 1.3 (Complex Numbers)

Q#1 Factorize

(i) a^2+4b^2

$a^2-4b^2(-1)$

$a^2-4b^2(i^2)$ $\therefore i^2=-1$

$(a)^2-(2bi)^2$

Using Formula

$a^2-b^2=(a+b)(a-b)$

$(a+2bi)(a-2bi)$ Ans

(ii) $9a^2+16b^2$

$9a^2-16b^2(-1)$

$9a^2-16b^2(i^2)$

$(3a)^2-(4bi)^2$

$(3a+4bi)(3a-4bi)$ Ans

(iii) $3x^2+3y^2$

$3(x^2+y^2)$

$3[x^2-y^2(-1)]$

$3[x^2-y^2i^2]$

$3[(x)^2-(yi)^2]$

$3(x+yi)(x-yi)$ Ans.

(iv) $144x^2+225y^2$

$144x^2-225y^2(-1)$

$144x^2-225y^2i^2$

$(12x)^2-(15yi)^2$

$(12x+15yi)(12x-15yi)$

$3(4x+5yi)\,3(4x-5yi)$

$9(4x+5yi)(4x-5yi)$ Ans.

Q#1

(v) $Z^2 - 2iZ - 1$

$Z^2 - 2iZ + (-1)$

$Z^2 - 2iZ + i^2 \quad \because i^2 = -1$

$(Z - i)^2$

$(Z - i)(Z - i)$ Ans.

(vi)

$Z^2 + 6Z + 13$

$Z^2 + 6Z + 9 + 4$

$(Z^2 + 6Z + 9) + 4$

$(Z+3)^2 - 4(-1)$

$(Z+3)^2 - 4i^2$

$(Z+3)^2 - (2i)^2$

$(Z+3+2i)(Z+3-2i)$ Ans.

(vii) $Z^2 + 4Z + 5$

$Z^2 + 4Z + 4 + 1$

$(Z^2 + 4Z + 4) - 1(-1)$

$(Z+2)^2 - i^2$

$(Z+2)^2 - (i)^2$

$(Z+2+i)(Z+2-i)$ Ans

(viii) $2Z^2 - 22Z + 65$

$2\left[Z^2 - 11Z + \frac{65}{2}\right]$ $\quad 11 \times \frac{1}{2} = \frac{11}{2}$

$2\left[Z^2 - 11Z + \left(\frac{11}{2}\right)^2 + \frac{65}{2} - \left(\frac{11}{2}\right)^2\right]$

$2\left[\left(Z - \frac{11}{2}\right)^2 + \left(\frac{65}{2} - \frac{121}{4}\right)\right]$

$2\left[\left(Z - \frac{11}{2}\right)^2 + \left(\frac{130 - 121}{4}\right)\right]$

$2\left[\left(Z - \frac{11}{2}\right)^2 + \frac{9}{4}\right]$

$2\left[\left(Z - \frac{11}{2}\right)^2 - \frac{9}{4}(-1)\right]$

$2\left[\left(Z - \frac{11}{2}\right)^2 - \left(\frac{3i}{2}\right)^2\right]$

$2\left[\left(Z - \frac{11}{2} + \frac{3i}{2}\right)\left(Z - \frac{11}{2} - \frac{3i}{2}\right)\right]$ Ans

Ex# 1.3

Q#2 Factorize the polynomial into linear factors.

$ax+b$

(i) Z^3+8

$(Z)^3+(2)^3$

Formula

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$(Z+2)[Z^2-2Z+4]$

Solving Z^2-2Z+4

$a=1 \quad b=-2, \; c=4$

$$Z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$Z=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(4)}}{2\times1}$$

$$Z=\frac{2\pm\sqrt{4-16}}{2}$$

$$Z=\frac{2\pm\sqrt{-12}}{2}$$

$i=\sqrt{-1}$

$$Z=\frac{2\pm\sqrt{12}\,i}{2}$$

$$Z=\frac{2\pm2\sqrt{3}\,i}{2}$$

$$Z=\frac{2(1\pm\sqrt{3}\,i)}{2}$$

$Z=1\pm\sqrt{3}\,i \Rightarrow Z-(1\pm\sqrt{3}\,i$

NOW $(Z+2)(1\pm\sqrt{3}\,i)$ OR

$(Z+2)(1+\sqrt{3}\,i)(1-\sqrt{3}\,i)$ Ar

$=(Z+2)[Z-(1+\sqrt{3}\,i)][Z-(1-\sqrt{3}\,i$

Q#3(ii) Ex# 1.3

$Z^3 + 27$

$(Z)^3 + (3)^3$

$(Z+3)(Z^2 - 3Z + 9)$

Solving $Z^2 - 3Z + 9$

Here $a = 1 \quad b = -3 \quad c = 9$

$$Z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Z = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2 \times 1}$$

$$Z = \frac{3 \pm \sqrt{9 - 36}}{2}$$

$$Z = \frac{3 \pm \sqrt{-27}}{2}$$

$\because i = \sqrt{-1}$

$$Z = \frac{3 \pm \sqrt{27}i}{2}$$

$$Z = \frac{3 \pm 3\sqrt{3}i}{2}$$

$$Z - \left(\frac{3 \pm 3\sqrt{3}i}{2}\right)$$

Now

$$(Z+3)\left[Z - \left(\frac{3 + \sqrt{3}i}{2}\right)\right]\left[Z - \left(\frac{3 - \sqrt{3}i}{2}\right)\right]$$

(iii) $Z^3 - 2Z^2 + 16Z - 32$

$Z^2(Z-2) + 16(Z-2)$

$(Z^2 + 16)(Z-2)$

$[Z^2 - 16(-1)](Z-2)$

$[Z^2 - 16i^2](Z-2)$

$[(Z)^2 - (4i)^2](Z-2)$

$(Z+4i)(Z-4i)(Z-2)$ Ans

Q#2 (iv)

$$Z^4 + 21Z^2 - 100$$
$$Z^4 + 25Z^2 - 4Z^2 - 100$$
$$Z^2(Z^2+25) - 4(Z^2+25)$$
$$(Z^2+25)(Z^2-4)$$
$$[Z^2 - 25(-1)](Z+2)(Z-2)$$
$$[Z^2 - 25i^2](Z+2)(Z-2)$$
$$[(Z)^2 - (5i)^2](Z+2)(Z-2)$$
$$(Z+5i)(Z-5i)(Z+2)(Z-2) \text{ Ans.}$$

(v) $Z^4 - 16$

$$(Z^2)^2 - (4)^2$$
$$(Z^2+4)(Z^2-4)$$
$$[Z^2 - 4(-1)][(Z)^2 - (2)^2]$$
$$(Z^2 - 4i^2)(Z+2)(Z-2)$$
$$(Z)^2 - (2i)^2 (Z+2)(Z-2)$$
$$(Z+2i)(Z-2i) \text{ and}$$
$$(Z+2)(Z-2) \text{ Ans.}$$

(vi) Z^4+3Z^2-4

$Z^4+4Z^2-Z^2-4$

$Z^2(Z^2+4)-1(Z^2+4)$

$(Z^2-1)(Z^2+4)$

$[(Z)^2-(1)^2][Z^2-4(-1)]$

$(Z+1)(Z-1)[Z^2-4i^2]$

$(Z+1)(Z-1)[(Z)^2-(2i)^2]$

$(Z+1)(Z-1)(Z-2i)(Z+2i)$ Ans.

(vii) Z^4+5Z^2+6

$Z^4+3Z^2+2Z^2+6$

$Z^2(Z^2+3)+2(Z^2+3)$

$(Z^2+3)(Z^2+2)$

$[Z^2-3(-1)][Z^2-2(-1)]$

$(Z^2-3i^2)(Z^2-2i^2)$

$[(Z)^2-(\sqrt{3}i)^2][(Z)^2-(\sqrt{2}i)^2]$

$(Z+\sqrt{3}i)(Z-\sqrt{3}i)(Z+\sqrt{2}i)(Z-\sqrt{2}i)$ Ans

(viii) Z^4+7Z^2-144

$Z^4+16Z^2-9Z^2-144$

$Z^2(Z^2+16)-9(Z^2+16)$

$(Z^2+16)(Z^2-9)$

$[Z^2-16(-1)][(Z)^2-(3)^2]$

$[Z^2-16i^2][(Z)^2-(3)^2]$

$[(Z)^2-(4i)^2][(Z)^2-(3)^2]$

$(Z+4i)(Z-4i)(Z+3)(Z-3)$ Ans

Q#3 Find the roots of $Z^4+7Z^2-144=0$ and hence express it as a product of linear factors.

$Z^4+7Z^2-144=0$

$Z^4+16Z^2-9Z^2-144=0$

$Z^2(Z^2+16)-9(Z^2+16)=0$

$(Z^2-9)(Z^2+16)=0$

$Z^2-9=0$ and $Z^2+16=0$

$Z^2=9$

$\sqrt{Z^2}=\pm\sqrt{9}$

$\boxed{Z=\pm 3}$

$Z^2=-16$

$\sqrt{Z^2}=\pm\sqrt{-16}$

$\boxed{Z=\pm 4i}$

S.S $\{\pm 3, \pm 4i\}$ roots

product as a linear factors.

$[(Z)^2-(3)^2][Z^2-16(-1)]$

$(Z+3)(Z-3)[Z^2-16i^2]$

$(Z+3)(Z-3)[(Z)^2-(4i)^2]$

$(Z+3)(Z-3)(Z+4i)(Z-4i)$

Ex # 1.3

Q # 4 Solve the complex quadratic equation by Completing Square method:-

(i) $2Z^2 - 3Z + 4 = 0$

Dividing by 2 on Both Sides

$$\frac{2Z^2}{2} - \frac{3}{2}Z + \frac{4}{2} = \frac{0}{2}$$

$$Z^2 - \frac{3}{2}Z + 2 = 0$$

$$Z^2 - \frac{3}{2}Z = -2 \qquad \frac{-3}{2} \times \frac{1}{2} = \frac{-3}{4}$$

$$Z^2 - \frac{3}{2}Z + \left(\frac{3}{4}\right)^2 = -2 + \left(\frac{3}{4}\right)^2$$

$$\left(Z - \frac{3}{4}\right)^2 = \frac{-2}{1} + \frac{9}{16}$$

$$\left(Z - \frac{3}{4}\right)^2 = \frac{-32 + 9}{16}$$

$$\left(Z - \frac{3}{4}\right)^2 = \frac{-23}{16}$$

$$\sqrt{\left(Z - \frac{3}{4}\right)^2} = \pm\sqrt{\frac{-23}{16}}$$

$$Z - \frac{3}{4} = \pm \frac{\sqrt{23}\,i}{4}$$

$$Z = \frac{3}{4} \pm \frac{\sqrt{23}\,i}{4}$$

$$Z = \frac{3 \pm \sqrt{23}\,i}{4}$$

$$S.S = \left\{\frac{3 \pm \sqrt{23}\,i}{4}\right\}$$

Q#4

(ii) $Z^2 - 6Z + 30 = 0$

$Z^2 - 6Z = -30$ $\quad 6 \times \frac{1}{2} = 3$

$Z^2 - 6Z + (3)^2 = (3)^2 - 30$

$(Z-3)^2 = 9 - 30$

$(Z-3)^2 = -21$

$\sqrt{(Z-3)^2} = \pm\sqrt{-21}$

$Z - 3 = \pm\sqrt{21}\, i$

$Z = 3 \pm \sqrt{21}\, i$

$S.S = \{3 \pm \sqrt{21}\, i\}$ Ans

(iii) $3Z^2 - 18Z + 50 = 0$

Dividing By 3 on both sides

$\frac{3}{3}Z^2 - \frac{18}{3}Z + \frac{50}{3} = \frac{0}{3}$

$Z^2 - 6Z + \frac{50}{3} = 0$ $\quad 6 \times \frac{1}{2} = 3$

$Z^2 - 6Z = -\frac{50}{3}$

$Z^2 - 6Z + (3)^2 = (3)^2 - \frac{50}{3}$

$(Z-3)^2 = 9 - \frac{50}{3}$

$(Z-3)^2 = \frac{27-50}{3} \Rightarrow \frac{-23}{3}$

$\sqrt{(Z-3)^2} = \pm\sqrt{\frac{-23}{3}}$

$Z - 3 = \pm\sqrt{\frac{23}{3}}\, i$

$Z = 3 \pm \sqrt{\frac{23}{3}}\, i$ Ans

(iv) $Z^2 + 4Z + 13 = 0$

$$Z^2 + 4Z = -13$$

$$Z^2 + 4Z + (2)^2 = (2)^2 - 13$$

$$(Z+2)^2 = 4 - 13 \qquad 4 \times \frac{1}{2} = 2$$

$$(Z+2)^2 = -9$$

$$\sqrt{(Z+2)^2} = \pm\sqrt{-9}$$

$$Z + 2 = \pm 3i$$

$$Z = -2 \pm 3i$$

$$S.S \; \{-2 \pm 3i\} \text{ Ans.}$$

(v) $2Z^2 + 6Z + 9 = 0$

Dividing by 2 on Both sides.

$$\frac{2}{2}Z^2 + \frac{6}{2}Z + \frac{9}{2} = \frac{0}{2}$$

$$Z^2 + 3Z + \frac{9}{2} = 0 \qquad 3 \times \frac{1}{2}$$

$$Z^2 + 3Z = -\frac{9}{2}$$

$$Z^2 + 3Z + (3/2)^2 = \left(\frac{3}{2}\right)^2 - \frac{9}{2}$$

$$\left(Z + \frac{3}{2}\right)^2 = \frac{9}{4} - \frac{9}{2}$$

$$\left(Z + \frac{3}{2}\right)^2 = \frac{9 - 18}{4}$$

$$(Z + 3/2)^2 = -\frac{9}{4}$$

$$\sqrt{(Z + 3/2)^2} = \pm\sqrt{-9/4}$$

$$Z + 3/2 = \pm\, 3/2\, i$$

$$Z + 3/2 = \pm 3/2\, i$$

$$Z = -\frac{3}{2} \pm \frac{3i}{2}$$

$$Z = \frac{-3 \pm 3i}{2}$$

$$Z = \frac{3(-1 \pm i)}{2}$$

$$S.S \left\{ 3\left(\frac{-1 \pm i}{2}\right) \right\}$$

(vi) $3Z^2 - 5Z + 7 = 0$

Dividing By 3 on Both sides.

$$\frac{3}{3}Z^2 - \frac{5}{3}Z + \frac{7}{3} = \frac{0}{3}$$

$$Z^2 - 5/3\, Z + 7/3 = 0$$

$\frac{5}{3} \times \frac{1}{2} = \frac{5}{6}$

$$Z^2 - 5/3\, Z = -7/3$$

$$Z^2 - 5/3\, Z + \left(\frac{5}{6}\right)^2 = \left(\frac{5}{6}\right)^2 - \frac{7}{3}$$

$$(Z - 5/6)^2 = \frac{25}{36} - \frac{7}{3}$$

$$(Z - 5/6)^2 = \frac{25 - 84}{36}$$

$$(Z - 5/6)^2 = \frac{-59}{36}$$

$$\sqrt{(Z - 5/6)^2} = \pm \sqrt{\frac{-59}{36}}$$

$$(Z - 5/6) = \pm \frac{\sqrt{59}\, i}{6}$$

$$Z = \frac{5}{6} \pm \frac{\sqrt{59}}{6} i$$

$$Z = \frac{5 \pm \sqrt{59}\, i}{6}$$

$$S.S \left\{ \frac{5 \pm \sqrt{59}\, i}{6} \right\} \text{Ans}$$

(iv) $Z^3 - 5Z^2 + Z - 5 = 0$

$Z^2(Z-5) + 1(Z-5) = 0$

$(Z-5)(Z^2+1) = 0$

$Z - 5 = 0 \; ; \; Z^2 + 1 = 0$

$\boxed{Z = 5} \; ; \; Z^2 = -1$

$\sqrt{Z^2} = \pm\sqrt{-1}$

$\boxed{Z = \pm i}$

$S.S = \{5, \pm i\}$ Ans.

(v) $4Z^4 - 25Z^2 + 21 = 0$

8.7

$4Z^4 - 21Z^2 - 4Z^2 + 21 = 0$

$Z^2(4Z^2 - 21) - 1(4Z^2 - 21) = 0$

$(4Z^2 - 21)(Z^2 - 1) = 0$

$4Z^2 - 21 = 0$

$4Z^2 = 21$

$Z^2 = \frac{21}{4}$

$\sqrt{Z^2} = \pm\sqrt{\frac{21}{4}}$

$\boxed{Z = \pm \frac{\sqrt{21}}{2}}$

$Z^2 - 1 = 0$

$Z^2 = 1$

$\sqrt{Z^2} = \pm\sqrt{1}$

$Z = \pm 1$

$S.S \left\{ \pm \frac{\sqrt{21}}{2}, \pm 1 \right\}$ Ans.

(vi) $Z^3 + Z^2 + Z + 1 = 0$

$Z^2(Z+1) + 1(Z+1) = 0$

$(Z+1)(Z^2+1) = 0$

$Z + 1 = 0$

$\boxed{Z = -1}$

$Z^2 + 1 = 0$

$Z^2 = -1$

$\sqrt{Z^2} = \pm\sqrt{-1}$

$Z = \pm i$

S.S $\{-1, \pm i\}$ Ans

Q#5 Solve the Equations

(i) $2Z^4 - 32 = 0$

$2(Z^4 - 16) = 0$

$Z^4 - 16 = 0/2$

$Z^4 - 16 = 0$

$(Z^2)^2 - (4)^2 = 0$

$(Z^2+4)(Z^2-4) = 0$

$Z^2+4 = 0 \; ; \; Z^2 - 4 = 0$

$Z^2 = -4 \; ; \; Z^2 = 4$

$\sqrt{Z^2} = \pm\sqrt{-4} \Rightarrow \pm 2i$

$\sqrt{Z^2} = \pm\sqrt{4} \Rightarrow \pm 2$

$S.S = \{\pm 2i, \pm 2\}$ Ans.

(ii) $3Z^5 - 243Z = 0$

$3Z(Z^4 - 81) = 0$

$3Z[(Z^2)^2 - (9)^2] = 0$

$3Z[(Z^2+9)(Z^2-9)] = 0$

$3Z(Z^2+9)(Z^2-9) = 0$

$3Z = 0$	$Z^2+9=0$	$Z^2-9=0$
$Z = \frac{0}{3}$	$Z^2 = -9$	$Z^2 = 9$
	$\sqrt{Z^2} = \pm\sqrt{-9}$	$\sqrt{Z^2} = \pm\sqrt{9}$
$\boxed{Z=0}$	$\boxed{Z=\pm 3i}$	$\boxed{Z=\pm 3}$

$S.S = \{0, \pm 3i, \pm 3\}$ Ans.

~~(iii) $3Z^2 - 18Z + 30$~~

(iii) $5Z^5 - 5Z = 0$

$5Z(Z^4 - 1) = 0$

$5Z[(Z^2)^2 - (1)^2] = 0$

$5Z(Z^2+1)(Z^2-1) = 0$

$5Z = 0$	$Z^2+1=0$	$Z^2-1=0$
$Z = 0/5$	$Z^2 = -1$	$Z^2 = 1$
$\boxed{Z=0}$	$\sqrt{Z^2} = \pm\sqrt{-1}$	$\sqrt{Z^2} = \pm\sqrt{1}$
	$Z = \pm 1i$	$\boxed{Z=\pm 1}$
	$\boxed{Z=\pm i}$	

$S.S = \{0, \pm i, \pm 1\}$ Ans

Q#6 Find a polynomial of degree 3 with zero 3, $-2i$, $2i$ and satisfying $p(1) = 20$.

$z_0 = 3 \quad z_1 = -2i \quad z_2 = 2i$

polynomial of degree 3

$$p(z) = a(z - z_0)(z - z_1)(z - z_2)$$

$$= a(z-3)(z+2i)(z-2i)$$

$$= a(z-3)[(z)^2 - (2i)^2]$$

$$= a(z-3)(z^2 - 4i^2)$$

$$P(z) = a(z-3)(z^2+4) \quad \text{—(i)}$$

~~put~~ $p(1) = 20$ Given

put $z = 1$

$$p(1) = a(1-3)(1+4)$$

$$20 = a(-2)(5)$$

$$-10a = 20$$

$$a = \frac{20}{-10} \Rightarrow -2$$

put $a = -2$ in (i)

$$p(z) = -2(z-3)(z^2+4)$$

$$p(z) = (-2z+6)(z^2+4)$$

$$= -2z^3 - 8z + 6z^2 + 24$$

$$= -2z^3 + 6z^2 - 8z + 24$$

Q #7

$Z_0 = 2i, Z_1 = -2i, Z_2 = 1$
$Z_3 = -i$

the polynomial of degree 4 is

$P(Z) = a(Z - Z_0)(Z - Z_1)(Z - Z_2)(Z - Z_3)$

$P(Z) = a(Z - 2i)(Z + 2i)(Z - 1)(Z + 1)$

$= a[(Z^2) - (2i)^2][Z^2 - (1)^2]$

$= a(Z^2 - 4i^2)(Z^2 - 1) \Rightarrow a(Z^2 + 4)(Z^2 - 1)$

$P(Z) = a(Z^4 - Z^2 + 4Z^2 - 4)$ —— (i)

Given that $P(2) = 240$

put $Z = 2$

$P(2) = a[(2)^4 - (2)^2 + 4(2)^2 - 4]$

$= a[16 - 4 + 16 - 4]$

$P(2) = 24a$

We know that

$P(2) = 240$

$24a = 240$

$a = \frac{240}{24} = 10$

put $a = 10$ in (i)

$p(Z) = 10(Z^4 + 3Z^2 - 4)$

$p(Z) = 10Z^4 + 30Z^2 - 40$

Q#8 Find a polynomial of degree 4 with Zeros $4, -4, (1+i), (1-i)$ and satisfying $p(2) = 72$

$Z_0 = 4 \quad Z_1 = -4, Z_2 = 1+i, Z_3 = 1-i$

the polynomial of degree 4 is

$p(Z) = a(Z - Z_0)(Z - Z_1)(Z - Z_2)(Z - Z_3)$

$= a(Z-4)(Z+4)(Z-1-i)(Z-1+i)$

$= a[(Z)^2 - (4)^2][\{(Z-1) - i\}\{(Z-1) + i\}]$

$= a(Z^2 - 16)[(Z-1)^2 - (i)^2]$

$= a(Z^2 - 16)[(Z-1)^2 + 1]$

$p(Z) = a(Z^2 - 16)(Z^2 + 1 - 2Z + 1)$

$P(Z) = a(Z^2-16)(Z^2+2-2Z)$ —— (i)

put $Z=2$

$P(2) = a[(2)^2-16][(2)^2+2-2(2)]$

$= a\,[4-16][4+2-4]$

$P(2) = a(-12)(2)$

$-24a = P(2)$

$-24a = 72$

$a = \frac{72}{-24} = -3$

put $a=-3$ in (i)

$= -3(Z^2-16)(Z^2+2-2Z)$

$-3(Z^4+2Z^2-2Z^3-16Z^2-32+32Z)$

$-3(Z^4-2Z^3-14Z^2+32Z-32)$

$= -3Z^4+6Z^3+42Z^2-96Z+96$

Ans

Comp