EX# 1·4

Q#2 Find the fourth root of 16, 81, 625. Also show that their sum is zero in each case.

(i) 16

Let x be the fourth root of 16

$$x^4 = 16$$
$$x^4 - 16 = 0$$
$$(x^2)^2 - (4)^2 = 0$$
$$(x^2+4)(x^2-4) = 0$$

$$x^2 + 4 = 0 \quad ; \quad x^2 - 4 = 0$$
$$x^2 = -4 \quad ; \quad x^2 = 4$$
$$\sqrt{x^2} = \pm\sqrt{-4} \quad ; \quad \sqrt{x^2} = \pm\sqrt{4}$$

$$x = \pm 2i \quad ; \quad x = \pm 2$$

Sum $= 2i + (-2i) + 2 + (-2) = 0$

(iii) 625

Let x be the fourth root of 625

$$x^4 = 625$$
$$x^4 - 625 = 0$$

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$(x^2)^2 - (25)^2 = 0$

$(x^2+25)(x^2-25) = 0$

$x^2+25=0$; $x^2-25=0$

$x^2 = -25$; $x^2 = 25$

$\sqrt{x^2} = \pm\sqrt{-25}$; $\sqrt{x^2} = \pm\sqrt{25}$

$\boxed{x = \pm 5i}$; $\boxed{x = \pm 5}$

Sum of all four fourth roots of unity is Zero

$5i + (-5i) + 5 + (-5) = 0$

Q#3 if $1, \omega, \omega^2$ are the Cube roots of unity, Show that $1+\omega^n+\omega^{2n} = 3$ Where n is a multiple of 3 respectively.

Sol:- $1+\omega^n+\omega^{2n}$ —— (i)

put $n=3$ in (i)

$\Rightarrow 1+\omega^3+\omega^{2(3)}$

$= 1+\omega^3+\omega^6$

$= 1+\omega^3+(\omega^3)^2$

$= 1+1+1$

$= 3$ Proved

Q#4 Evaluate :- (i) $\left(\dfrac{-1+\sqrt{-3}}{2}\right)^7 + \left(\dfrac{-1-\sqrt{-3}}{2}\right)^7$

We know that

$\omega = \dfrac{-1+\sqrt{-3}}{2}$ and $\omega^2 = \dfrac{-1-\sqrt{-3}}{2}$ so

$(\omega)^7 + (\omega^2)^7 \Rightarrow \omega^7 + \omega^{14}$

$= \omega \cdot \omega^6 + \omega^2 \cdot \omega^{12}$

$= \omega \cdot (\omega^3)^2 + \omega^2 \cdot (\omega^3)^4$

$= \omega + \omega^2$

$= -1$ Ans $(1+\omega+\omega^2=0)$.

(ii) $(-1+\sqrt{-3})^5 + (-1-\sqrt{-3})^5$

$\left[2\left(\dfrac{-1+\sqrt{-3}}{2}\right)\right]^5 + \left[2\left(\dfrac{-1-\sqrt{-3}}{2}\right)\right]^5$

$(2\omega)^5 + (2\omega^2)^5$

$32\omega^5 + 32\omega^{10}$

$32(\omega^2 \cdot \omega^3 + \omega^9 \cdot \omega)$

$32[\omega^2 \cdot (1) + (\omega^3)^3 \cdot \omega]$

$32(\omega^2 + \omega) \Rightarrow 32(-1)$

$= -32$ Ans

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Q # 5: Show that $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)(1-\omega^8+\omega^{16})\cdots$ to $2n$ factors $= 2^{2n}$

We Know that

$1+\omega+\omega^2=0$

$1+\omega^2=-\omega\;;\quad 1+\omega=-\omega^2$

$[(1+\omega^2)-\omega][1+\omega\cdot\omega^3-\omega^2][1+\omega^2\cdot\omega^6-\omega^4][1+\omega\cdot\omega^{15}-\omega^8]$

$(-\omega-\omega)(-\omega^2-\omega^2)(1+\omega^2\cdot(\omega^3)^2-\omega\,\omega^3)(1+\omega-\omega^2\cdot\omega^6)\cdots$ to $2n$ factors

$(-2\omega)(-2\omega^2)(1+\omega^2-\omega)(1+\omega-\omega^2)\cdots$ to $2n$ factors

$(-2\omega)(-2\omega^2)(-\omega-\omega)(-\omega^2-\omega^2)\cdots$ to $2n$ factors

$[(-2\omega)(-2\omega)][(-2\omega^2)(-2\omega^2)]\cdots$ to $2n$ factors

$[(-2\omega)^2]^n[(-2\omega^2)^2]^n \Rightarrow$

$= -2^{(2n)}\cdot\omega^{4n}\cdot(-2)^{(2n)}(\omega^2)^n$

$(-2)^{2n}\cdot(\omega)^{3n} \Rightarrow 2^{2n}\cdot(\omega^3)^n \Rightarrow 2^{2n}$

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Q#6 Prove that $\left(\dfrac{i+\sqrt{3}}{2}\right)^8 + \left(\dfrac{i-\sqrt{3}}{2}\right)^8 = -1$

$$\left[\frac{1}{i}\left(\frac{-i^2+i\sqrt{3}}{2}\right)\right]^8 + \left[\frac{1}{i}\left(\frac{i^2-i\sqrt{3}}{2}\right)\right]^8$$

$$\left[\frac{1}{i}\left(\frac{-1+\sqrt{-1}\sqrt{3}}{2}\right)\right]^8 + \left[\frac{1}{i}\left(\frac{-1-\sqrt{-1}\sqrt{3}}{2}\right)\right]^8$$

$$\left[\frac{1}{i}\left(\frac{-1+\sqrt{-3}}{2}\right)\right]^8 + \left[\frac{1}{i}\left(\frac{-1-\sqrt{-3}}{2}\right)\right]^8$$

$$= \left(\frac{1}{i}\times\omega\right)^8 + \left(\frac{1}{i}\times\omega^2\right)^8$$

$$= \frac{\omega^8}{i^8} + \frac{\omega^{16}}{i^8}$$

$$\frac{\omega^2\cdot\omega^6}{(i^2)^8} + \frac{\omega\cdot\omega^{15}}{(i^2)^8}$$

$$= \frac{\omega^2\cdot(\omega^3)^2}{(-1)^8} + \frac{\omega\cdot(\omega^3)^5}{(-1)^8}$$

$$= \omega^2 + \omega$$

$$= -1$$ Proved

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Q#7 Evaluate $\sum_{K=0}^{5} \omega^{2K}$, where ω is an imaginary cube root of unity.

put $K = 0, 1, 2, 3, 4, 5$

ω^{2K}

$\omega^{2(0)} = \omega^{0} = 1$

$\omega^{2(1)} = \omega^{2}$

$\omega^{2(2)} = \omega^{4}$

$\omega^{2(3)} = \omega^{6}$

$\omega^{2(4)} = \omega^{8}$

$\omega^{2(5)} = \omega^{10}$

Taking Sum (Σ)

$= 1 + \omega^2 + \omega^4 + \omega^6 + \omega^8 + \omega^{10}$

$= 1 + \omega^2 + \omega\cdot\omega^3 + (\omega^3)^2 + \omega^2\cdot\omega^6 + \omega\cdot\omega^9$

$= 1 + \omega^2 + \omega + 1 + \omega^2\cdot(\omega^3)^2 + \omega\cdot(\omega^3)^3$

$= (1 + \omega + \omega^2) + (1 + \omega^2 + \omega)$

$= 0 + 0$ * $1 + \omega + \omega^2 = 0$

$= 0$ Ans

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Ex # 1.4

Q#8: if ω is an imaginary cube roots of unity prove that $\dfrac{a+b\omega^2+c\omega}{a\omega^2+b\omega+c}=\omega$

$$= \frac{\omega(a+b\omega^2+c\omega)}{(a\omega^3+b\omega^2+c\omega)}$$

$$= \frac{\omega(a+b\omega^2+c\omega)}{(a+b\omega^2+c\omega)}$$

$= \omega$ proved

Q#9: if ω is a cube root of unity, prove that

$$\frac{a\omega^{12}+b\omega^{17}+c\omega^{19}}{a\omega^{14}+b\omega^{22}+c\omega^{30}}=\omega$$

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$$\frac{a(\omega^3)^4 + b\omega^2\cdot\omega^{15} + c\omega\cdot\omega^{18}}{a\omega^2\cdot\omega^{12} + b\omega\cdot\omega^{21} + c(\omega^3)^{10}}$$

$$= \frac{a(1)^4 + b\omega^2(\omega^3)^5 + c\cdot\omega\cdot(\omega^3)^6}{a\cdot\omega^2\cdot(\omega^3)^4 + b\cdot\omega\cdot(\omega^3)^7 + c(1)^{10}}$$

$$= \frac{a + b\omega^2 + c\omega}{a\omega^2 + b\omega + c}$$

$$= \frac{\omega(a + b\omega^2 + c\omega)}{a\omega^3 + b\omega^2 + c\omega}$$

$$\frac{\omega(a + b\omega^2 + c\omega)}{(a + b\omega^2 + c\omega)}$$

$$= \omega$$ Proved

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