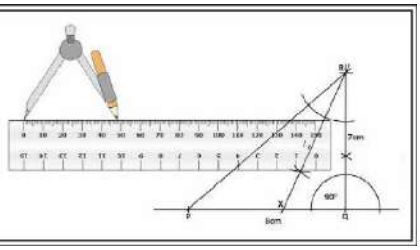
11 UNIT

LOCI AND CONSTRUCTION



Construction of Triangles:

A triangle is a closed figure having three sides and three angles. We construct triangle in the following cases:

1. When measure of all three sides are given.
2. When measure of two sides and their included angle are given.
3. When measure of one side and measure of two angles are given.
4. When measure of two sides and an angle opposite to one of them is given.

(b) Construction of a triangle when the measure of two sides and their included angle is given

Example 2:

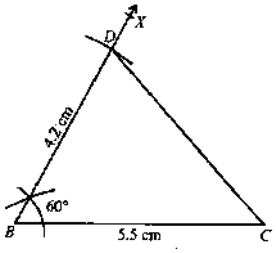
Construct a triangle BCD in which measures of two sides are 5.5 cm and 4.2 cm and measure of their included angle is 60°.

Solution:

Steps of construction

1. Draw a line segment BC of length 5.5cm.
2. Draw an angle 60° at point B using a pair of compasses and draw a ray $\overrightarrow{BX}$ through this angle.
3. Draw an arc of radius 4.2 cm with centre at point B intersecting $\overrightarrow{BX}$ at point D.
4. Join C and D.

Hence, $\bigcirc$ is the required triangle.



(c) Construction of a triangle when measure of one side and two angles is given

Example 3:

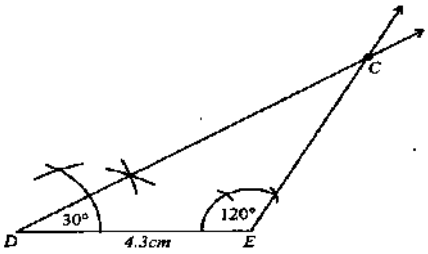
Draw a triangle CDE when $m\overline{DE} = 4.3\text{cm}$, $m\angle D = 30°$ and $m\angle E = 120°$.

Solution:

Steps of construction:

1. Draw $m\overline{DE} = 4.3\text{cm}$.
2. Draw angles 30° and 120° at points D and E respectively using a pair of compasses and draw two rays through these angles from D and E.
3. These two rays intersect each other at point C.

Hence, ΔCDE is the required triangle.



(d) Construction of a triangle when measure of two sides and angle opposite to one of them is given

Consider the given two cases:

- If the measure of one angle is greater than or equal to 90°.
- If the measure of angle is less than 90°.

Example 4:

Remember:

There are three types of triangles w.r.t. sides:

Scalene triangle: All sides are of different length.

Isosceles triangles: Two sides are of equal length.

Equilateral triangle: All sides of equal length.

There are three types of triangles w.r.t. angles:

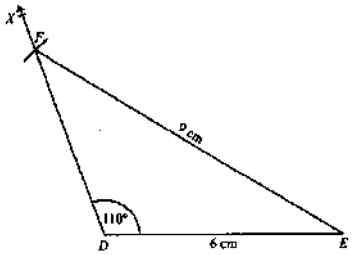
Acute angled triangle: All angles are of measure less than 90°.

Obtuse angled triangle: One angle is of measure greater than 90°.

Right angled triangle: One angle is of measure equal to 90°.

Construct a triangle DEF **when** $m\overline{DE} = 6\text{cm}$, $m\angle D = 110°$ **and**

$m\overline{EF} = 9\,\text{cm}$.



Steps of construction:

- Draw $m\overline{DE} = 6\text{cm}$.
- Construct $m\angle D = 110°$ using protractor and draw $\overrightarrow{DX}$ through this angle.
- Draw an arc of radius 9 cm with centre at point E intersecting $\overrightarrow{DX}$ at point F.
- Join E and F.

Hence, ΔDEF is the required triangle

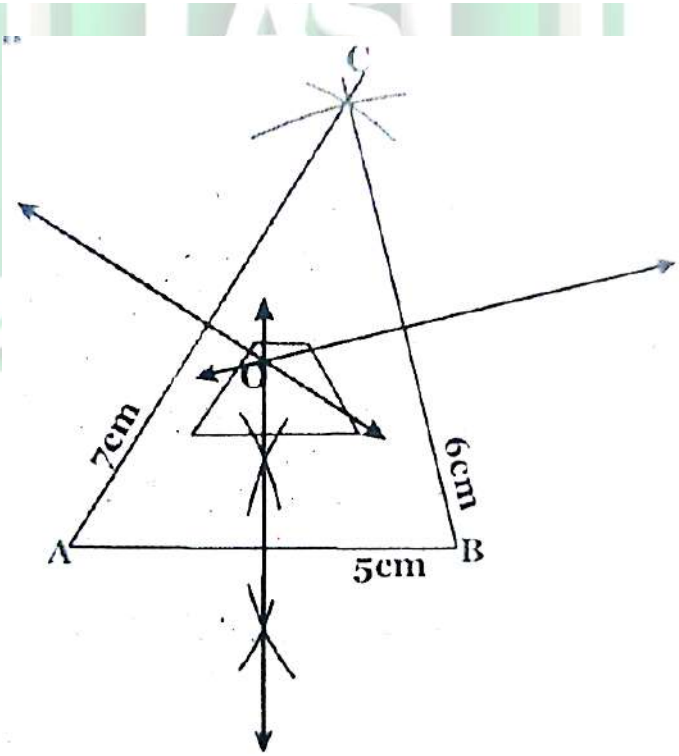
If the given angle opposite to the given is obtuse, only one triangle is possible.

EXERCISE 11.1

1. Construct ΔABC **with the given measurements and verify that the perpendicular bisectors of the triangle are concurrent.**

1. $m\overline{AB} = 5cm$, $m\overline{BC} = 6cm$ **and** $m\overline{AC} = 7cm$

Ans:

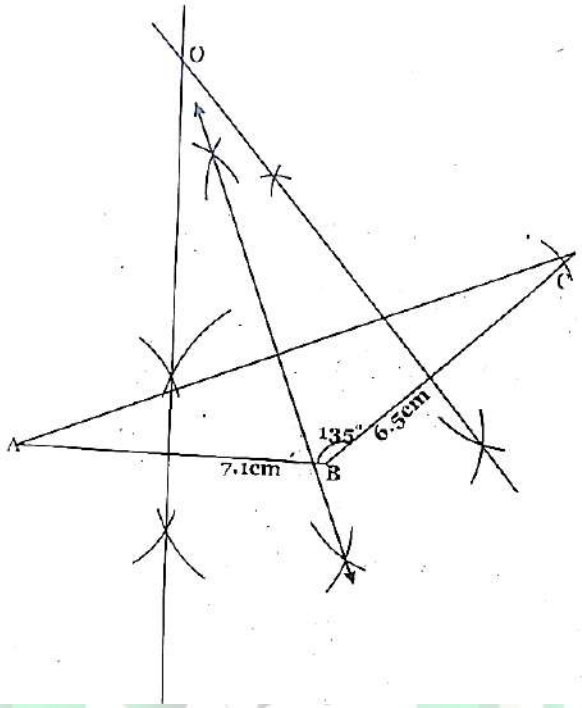


- Draw a line segment AC = 7cm
- Taking A as centre draw an arc 5cm
- Taking C as centre draw an arc 6cm

Hence ABC is required triangle.

2. $m\overline{AB} = 7.1cm$, $m\angle B = 135°$ **and** $m\overline{BC} = 6.5cm$

Ans:



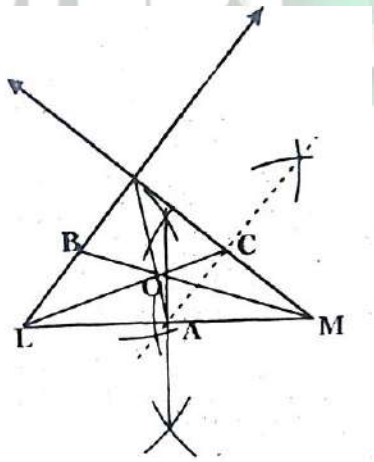
1. Draw a line segment $\overline{AB} = 7.1cm$
2. Taking B as centre draw an angle 135°
3. Taking B as centre draw an arc 6.5cm

Hence *ABC* is a triangle draw perpendicular bisector.

4. Construct ΔLMN of the following measurements and verify that the medians of the triangle are concurrent.

1. $m\overline{LM} = 4.9cm,\ m\angle L = 51°$ **and** $m\angle M = 38°$

Ans:

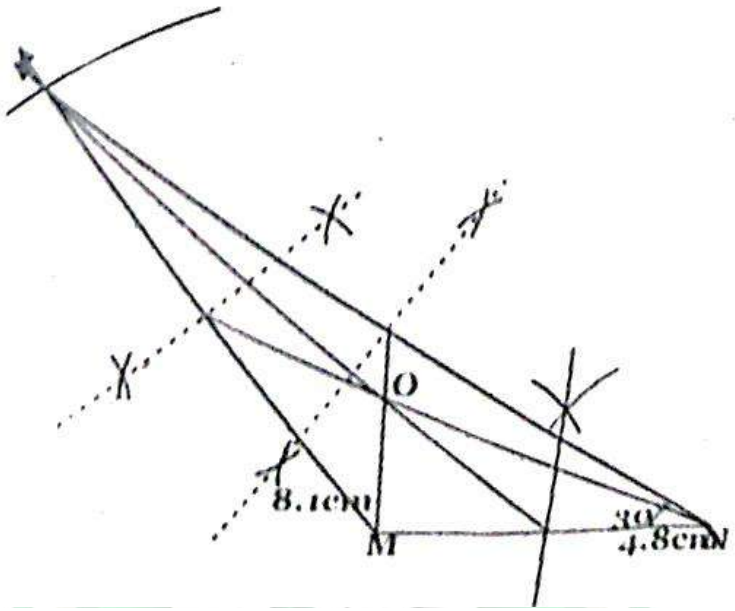


1. Draw line segment $\overline{LM} = 4.9cm$
2. Taking L as centre draw an angle of 51°
3. Thus LMN is the required triangle
4. Draw the median

5. $m\overline{MN} = 4.8cm,\ m\angle N = 30°$ **and** $m\overline{LM} = 8.1cm$

Ans:

1. Draw a line segment $\overline{MN} = 4.8cm$
2. Taking N as centre drawn angle of 30°
3. Taking M as centre drawn arc of 8.1 cm
4. Hence LMN is the required triangle
5. Draw the median

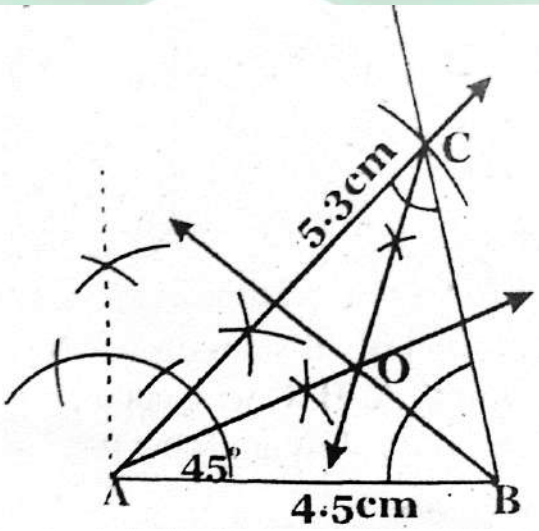


6. Verify that the angle bisectors of ΔABC are concurrent with the following measurement:

1. $m\overline{AB} = 4.5cm$, $m\angle A = 45°$ **and** $m\overline{AC} = 5.3cm$

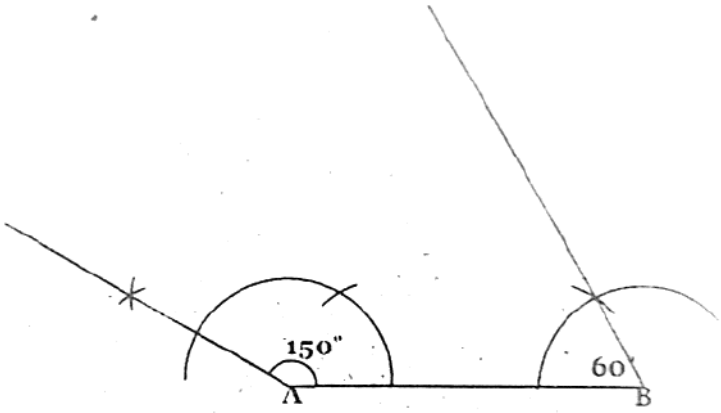
Ans:

1. Draw a line segment $\overline{AB} = 4.5cm$
2. Taking *A* as centre draw an angle 45°
3. Taking *A* as centre draw an arc 5.3 cm
4. Hence *ABC* is the required triangle
5. Draw the angle bisector



6. $m\overline{AB} = 6cm,\ m\angle A = 150°$ **and** $m\angle B = 60°$

Ans:

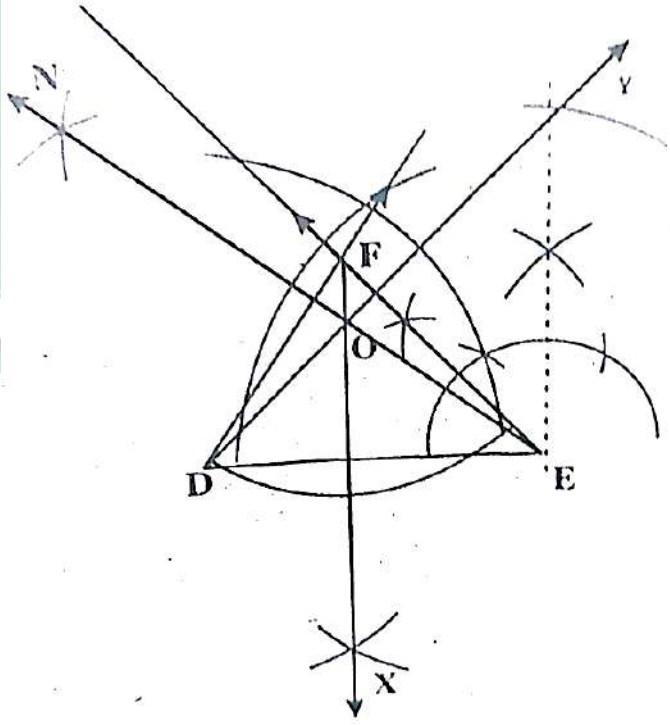


Not possible sum of three angle in a triangle is 180 but in this case sum of two angle is $150 + 60 = 210$

So, it is not possible

7. **Given the measurements of** $\Delta DEF : m\overline{DE} = 4.8cm,\ m\overline{EF} = 4cm$ **and** $m\angle E = 45°$, **draw altitudes of** ΔDEF **and find orthocentre.**

Ans:

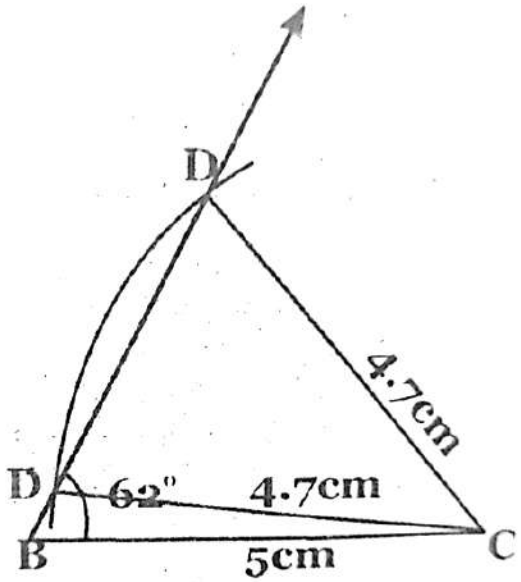


1. Draw a line segment $DE = 4.8cm$
2. Taking E as centre draw an angle of 45°
3. Taking E as centre draw arec of 4 cm for F
4. Hence DEF is the required triangle All the altitudes passes through same points O so altitudes of triangle are concurrent so point is orthocenter

5. Construct the following triangles and find whether there exists any ambiguous case.

1. $\Delta BCD; mBC = 5cm,\ m\angle B = 62°$ **and** $m\overline{CD} = 4.7cm$

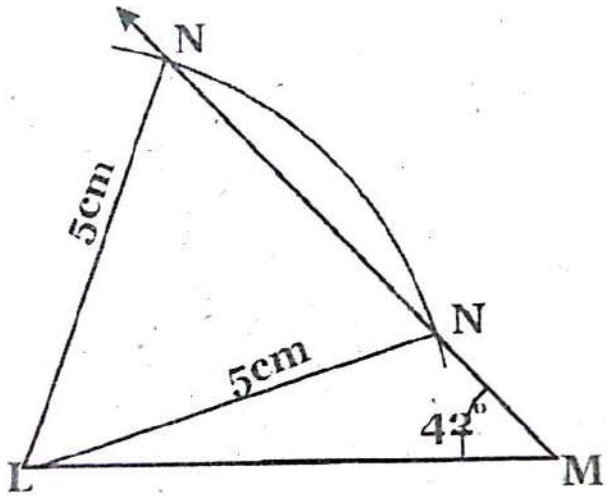
Ans:



1. Draw a line segment $\overline{BC} = 5cm$
2. Taking *B* as centre draw an angle of 62°
3. Taking C as centre draw an arc of 4.7 cm
4. Hence *BCD* is the required triangle

5. $\Delta KLM; m\overline{LM} = 6cm,\ m\angle M = 42°$ **and** $m\overline{LN} = 5cm$

Ans:



1. Draw a line segment $\overline{LM} = 6cm$
2. Taking *M* as centre draw an angle of 42°
3. Taking *L* as centre draw an arc of 5 cm
4. Hence *LMN* is the required triangle

9th Math

Loci and Construction

A locus (plural loci) is a set of points that follow a given rule. In geometry, loci are often used to define the positions of points relative to one another or to other geometric figures. Some common types of loci along with detailed explanations will be discussed.

Do you know? In Latin, the work *locus* is defined by the English term, location.

Remember!
Equidistant: Let A be a fixed point and B be a set of points. If A is at equal distance from all points of B, then A is said to be equidistant from B.

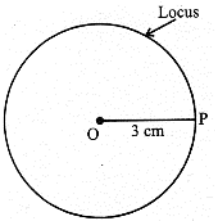
Loci in Two Dimension

We study the loci, circle, parallel lines, perpendicular bisector and angle bisector in two dimensions and apply them to real life situations.

Circle

The locus of a point whose distance is constant from a fixed point is called a circle.

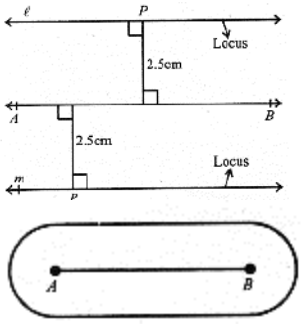
For example, the locus of a point P whose distance is 3 cm from a fixed-point O is a circle of radius 3 cm and centre at point O.



Parallel Lines

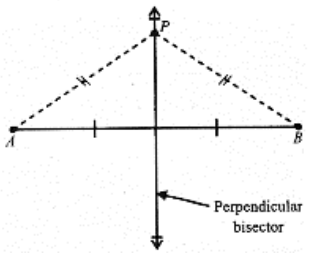
The locus of a point whose distance from a fixed line is constant are parallel lines, ℓ and m e.g. the locus of a point P whosEe distance is 2.5 cm from a fixed line AB are parallel lines at a distance of 2.5 cm from AB.

For example, a locus of points equidistant from a line segment creates a **sausage shape**. We can think of this type of locus as a track surrounding a line segment.



Perpendicular Bisector

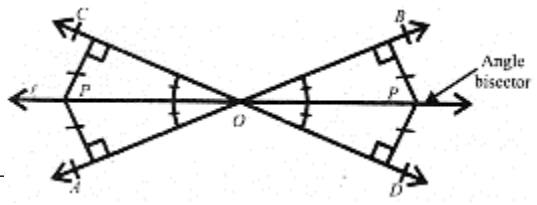
The locus of a point whose distance from two fixed points is constant is called a perpendicular bisector. For example, the locus of a point P whose distance from fixed points A and B is constant is the perpendicular bisector of the line segment AB.



Angle Bisector

The locus of a point whose distance is constant from two intersecting lines is called an angle bisector.

For example, the locus of a point P whose distance is constant from two lines AB and CD intersecting at O is the angle bisector (ℓ) of $\angle AOC$ and $\angle BOD$.



Remember:

1. Locus of points equidistant from a fixed point is a circle and equidistant from two fixed points is a perpendicular bisector.
2. Locus of points equidistant from a fixed line are two parallel lines and equidistant from two fixed intersecting lines is angle bisector.

Intersection of Loci

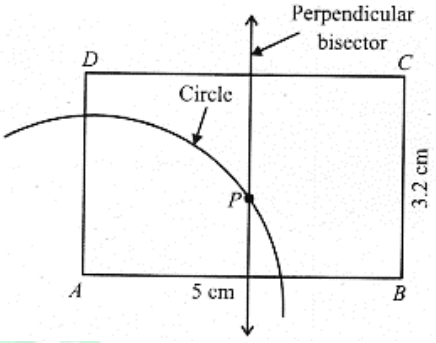
If two or more loci intersect at a point P, then P satisfies all given conditions of the loci. This will be explained in the following examples:

Example 10:

Construct a rectangle $ABCD$ with $mAB = 5\text{cm}$ and $mBC = 3.2\text{cm}$. Draw the locus of all points which are:

5. **at a distance of 3.1 cm from point A.**
6. **equidistant from A and B.**

Label the point P inside the rectangle which is 3.1 cm from point A and equidistant from A and B.



Solution:

Construct rectangle $ABCD$ with given lengths.

- Draw a circle of radius 3.1 cm with centre at A.
- Draw perpendicular bisector of AB.

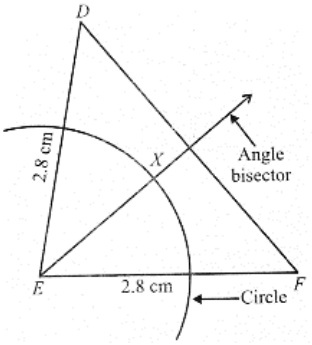
The two loci intersect at P inside the rectangle which is 3.1 cm from point A and equidistant from A and B.

Example 11:

Construct an isosceles triangle DEF with vertical angle $80°$ at E and $m\overline{EF} = m\overline{DE} = 4.8$ cm. Draw the locus of all points which are:

7. **at a distance of 2.8 cm from point E,**
8. **equidistant from $\overline{DE}$ and $\overline{EF}$.**

Label the point X inside the triangle which is 2.8 cm from point E and equidistant from $\overline{ED}$ and $\overline{EF}$.



Solution:

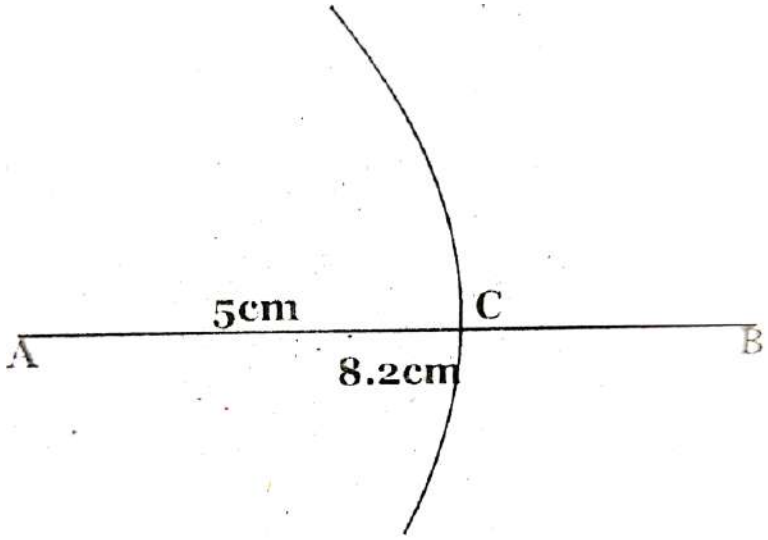
Construct triangle DEF with given measurements.

- Draw a circle of radius 2.8 cm with center at E.
- Draw angle bisector of angle DEF. The two loci intersect at X inside the triangle which is 2.8 cm from point E

EXERCISE 11.2

- **Two points A and B are 8.2 cm apart. Construct the locus of points 5 cm from point A.**

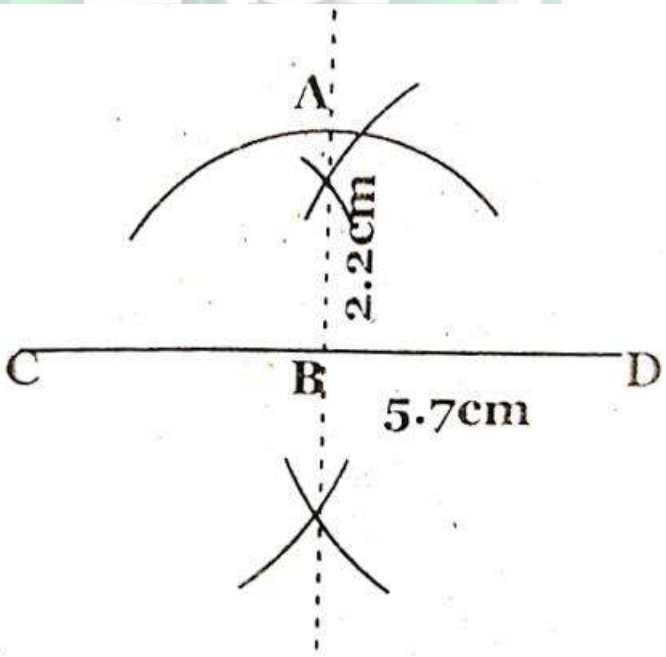
Ans:



- Draw a straight line segment such that $m\overline{AB} = 8.2cm$
- Take point A as center, draw an arc of radius 5cm which cuts line segment $\overline{AB}$ at point C. so arc represents locus of points 5cm from point A.

- **Construct a locus of point 2.2 cm from line segment CD of measure 5.7cm.** $\sin\theta$

Ans:

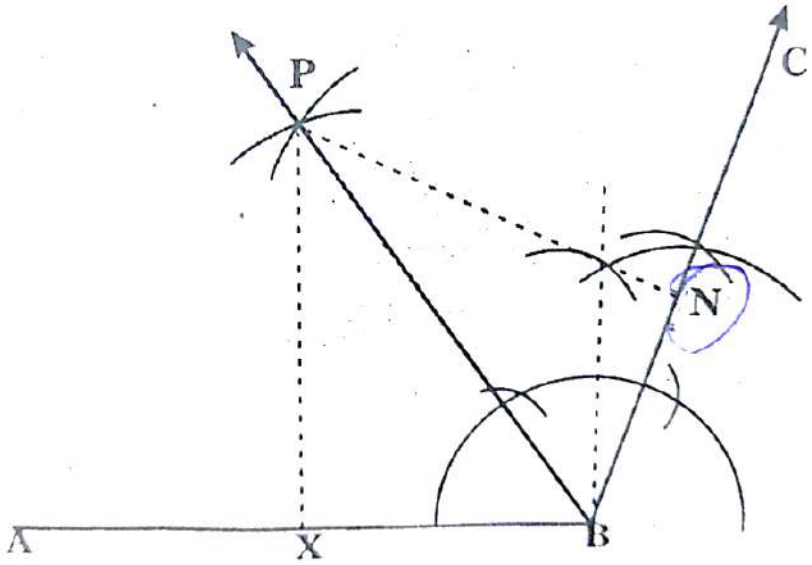


9. Draw a line segment $m\overline{CD} = 5.7cm$
10. Draw perpendicular bisector of $\overline{CD}$
11. $\overline{AB}$ is perpendicular bisector which meet $\overline{CD}$ at point B.
12. Take point B as centre, draw an arc of radius 2.2cm which cuts $\overline{AB}$ at *A*. which is the required locus of point 2.2cm from line segment $\overline{CD}$ of measure 5.7 cm

- **Construct an angle** $ABC = 105°$. **construct a locus of point P which moves such that it is**

equidistant from $\overline{BA}$ and $\overline{BC}$.

Ans:

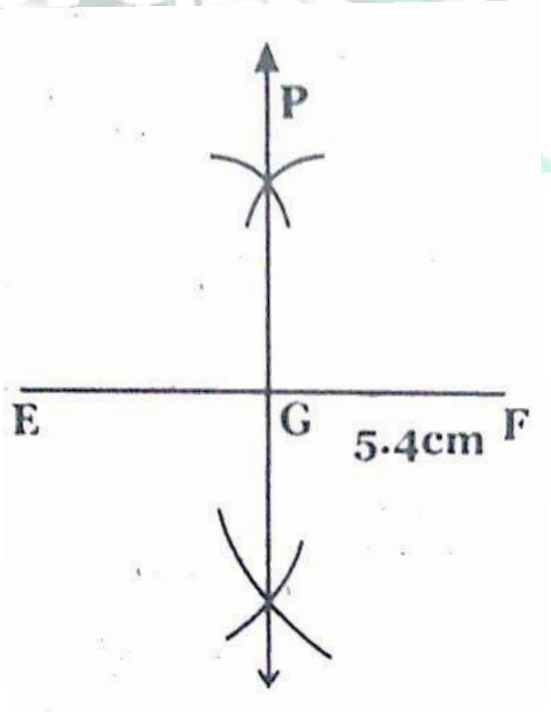


Point *P* lies on angle bisector of 105°.
Which is equidistant from its angle arm.
Here $\overline{PX} = \overline{PZ}$

- **Two points *E* and *F* are 5.4 cm apart. Construct a locus of a point *P* which moves such that it is equidistant from *E* and *F*.**

Ans:

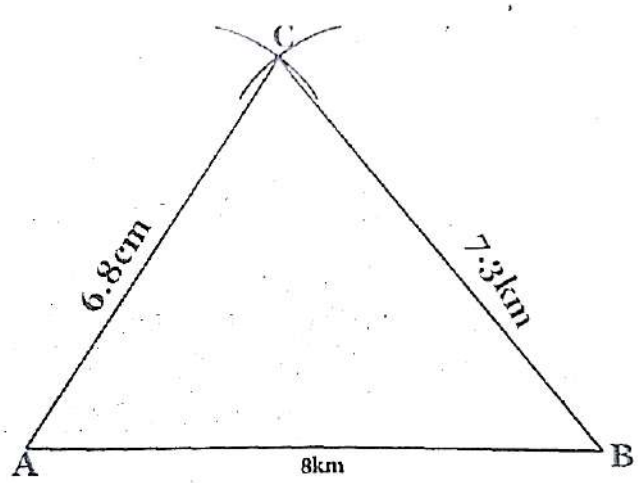
Locus of point *P* whose distance from fixed point *E* and *F* is constant is the perpendicular bisector of the line segment *EF*.



- **The island has two main cities A and B 8 km apart. Kashif lives on the island exactly 6.8 km form city A and exactly 7.3 km from city B. Mark with a cross the points on the island**

where Kashif could live.

Ans:



13. Draw a line segment $m\overline{AB} = 8km.$ here *A, B* represent two towns.
14. Mark an arc of radius 6.8km taking pint *A* as cente
15. Mark an arc of radius 7.3km taking point *B* as centre.
16. Both arcs intersect each other at point *C* where Kashif could lives.

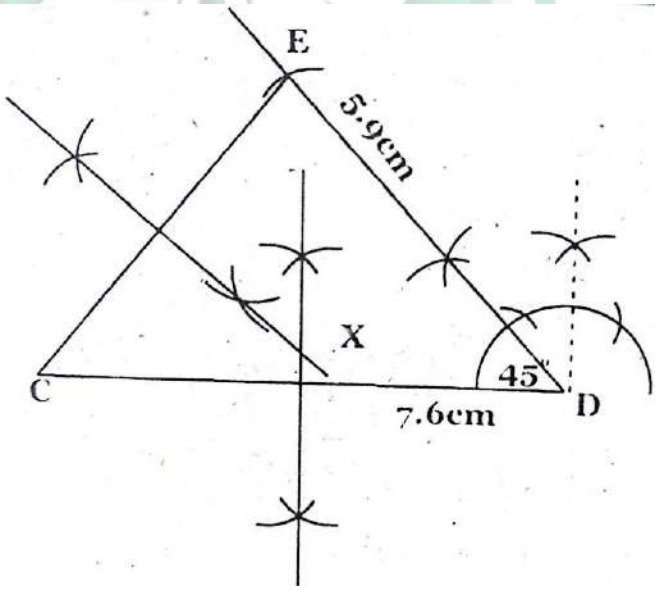
- **Construct a triangle CDE with** $m\overline{CD} = 7.6cm, m\angle D = 45°$ **and** $mDE = 5.9cm$

Draw the locus of all points which are:

1. Equidistant from *C* and *D*

2. Equidistant from $\overline{CD}$ and $\overline{CE}$

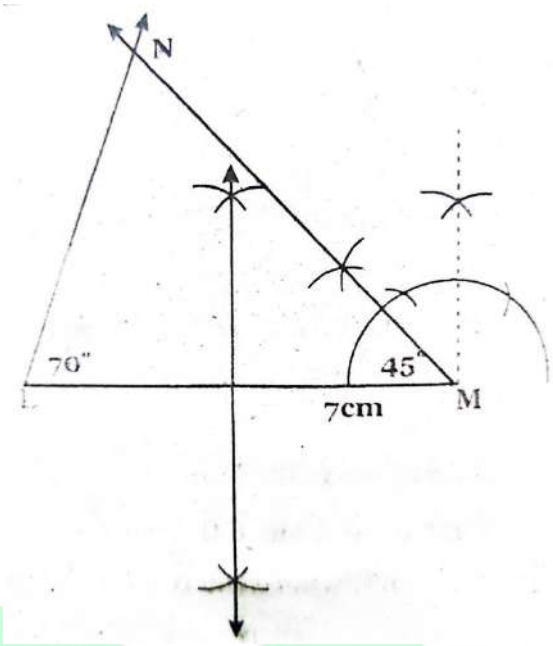
Ans:



3. Locus of all points which are equidistant from C and d is perpendicular bisector of $\overline{CD}$
4. Both loci intersect each other at point *X*.

- **Construct a triangle** *LMN* **with** $m\overline{LM} = 7cm, m\angle L = 70°$ **and** $m\angle M = 45°$. **Find a point within the triangle** *LMN* **which is equidistant form L and M and 3 cm from** *L*.

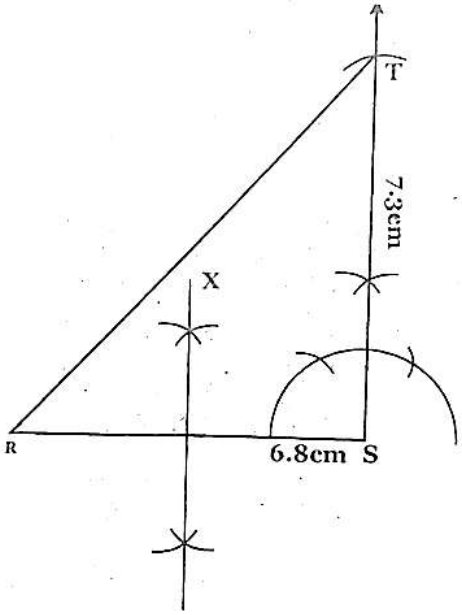
Ans:



We cannot find point within the triangle which is equidistant from L and M and 3 cm from *L*. See the diagram

- **Construct a right angled triangle RST with** $m\overline{RS} = 6.8cm, m\angle S = 90°$ **and** $m\overline{ST} = 7.5cm.$ **Find a pint within the triangle RST which is equidistant from** $\overline{RS}$ **and** $\overline{RT}$ **and 4.5 cm from *R*.**

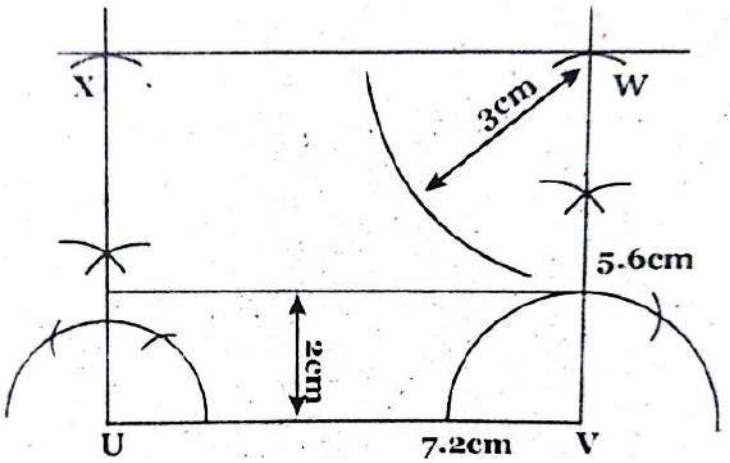
Ans:



Constrict right angled triangle *RST*. Draw perpendicular bisector of $\overline{RS}$.
Take point *R* as centre, mark an arc of radius 4.5cm which cuts perpendicular bisector at point *X*.

- **Construct a triangle** *CDE* **with** $m\overline{CD} = 7.6cm, m\angle D = 45°$ **and** $m\overline{DE} = 5.9cm$**. The locus of points at a distance of 2 cm from** $\overline{UV}$ **and 3.5 cm from** *W***.**

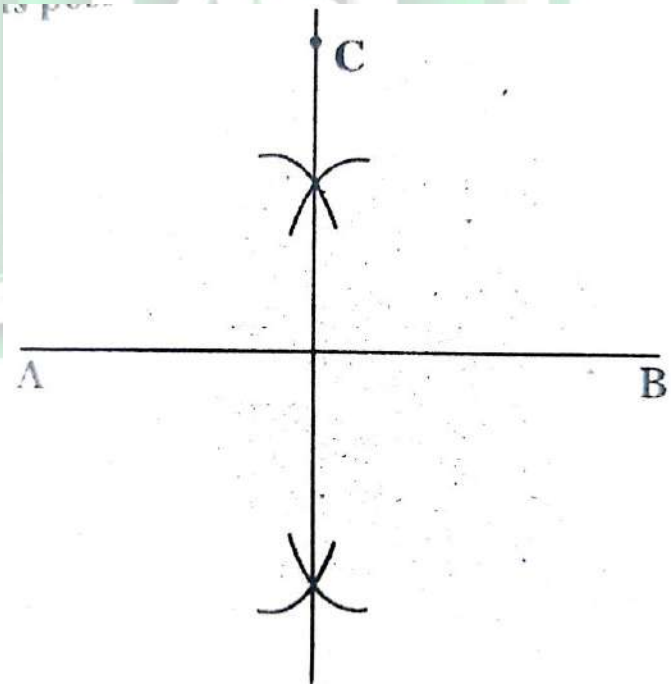
Ans:



5. Construct a rectangle *UVWX* according to measurement.
6. Draw a line parallel to *UV* at a distance 2 cm from $\overline{UV}$.
7. Take point *W* as centre, draw an arc of radius 3cm.

- **Imagine two cell towers located at points A and B on a coordinate plane. The GPS-enabled device, positioned somewhere on the plane, receives signals from both towers. To ensure accurate navigation, the device is placed equidistant form both towers to estimate its position. Draw this locus of navigation.**

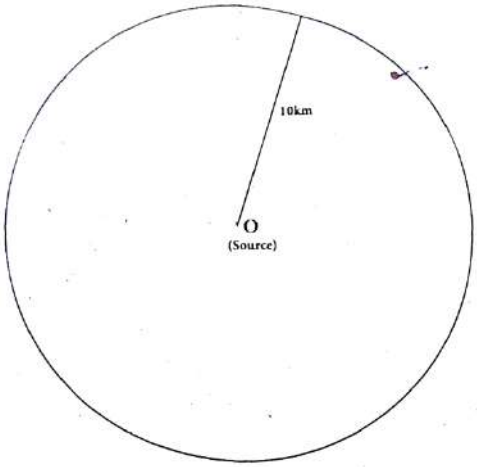
Ans:



Any point on perpendicular bisector is equidistant from end points of line segment.
So, locus of navigation is the perpendicular bisector of $\overline{AB}$. Point C is the position of device.

- **Epidemiologists use loci to determine infection zones, especially for contagious diseases, to predict the spread and take containment measures. In the case of a disease outbreak, authorities might determine a quarantine zone withing 10 km of the infection source. Draw the locus of all points 10 km from the source defining the quarantine area to monitor and control the disease's spread.**

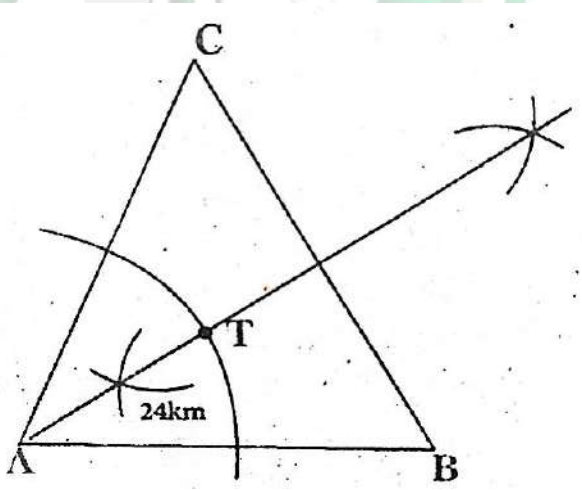
Ans:



In the diagram, let O be the infection source. We draw a circle of radius 10km (5cm) to determine quarantine zone which shows locus of all points 10km from source defining the quarantine area to monitor and control the spread of disease.

- **There is a treasure buried somewhere on the island. The treasure is 24 kilometres from A and equidistant from B and C. using a scale of 1cm to represent 10 km, find where the treasure could be buried.**

Ans:

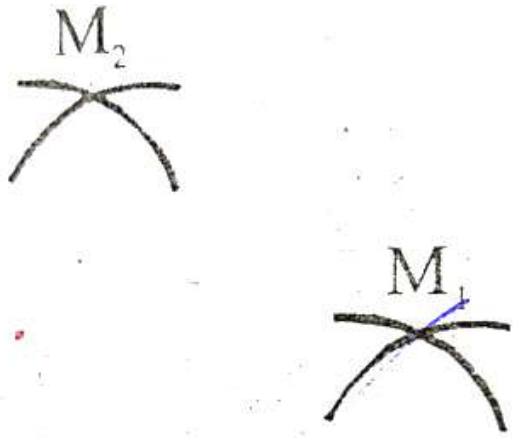


8. Draw ΔABC of any measurements.
9. Draw perpendicular bisector of side $\overline{BC}$
10. Draw locus of points 2.4km away from points A.
11. The point of intersection of locus and $\perp$ bisector given the point where treasure is buried.

- **There is an apple tee at a distance of 90 metres from banana tee in the garden of Sara's**

house. Sara wants to plant a mango tree M which is 64 metres from apple tee and between 54 and 82 metres from the banana tree. Using a scale of 1cm to represent 10m, Find the points where the mango tree should be planted.

Ans:



"A" denotes apple tree. "B" denotes banana tree Distance between apple tree and banana trees $=90m(9\text{cm})$.

By given conditions. M_1 and M_2 are points where mango trees should be planted.

REVIEW EXERCISE

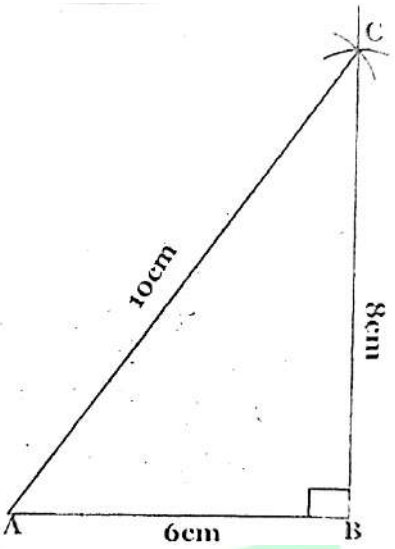
- **Four options are given against each statement. Encircle the correct one.**
- **A triangle can be constructed if the sum of the measure of any two sides is _____ the measure of the third side.**
  (a) Less than (b) Greater than
  (c) Equal to (d) Greater than and equal to
- **An equilateral triangle _____**
  (a) Can be isosceles
  (b) Can be right angled
  (c) Can be obtuse angled
  (d) Has each angle equal to 50°
- **If the sum of the measures of two angles is less than 90°, then the triangle is _____**
  (a) Equilateral
  (b) Acute angled
  (c) Obtuse angled (d) right angled
- **The line segment joining the midpoint of a side to its opposite vertex in a triangle is called _____**
  (a) Median
  (b) Perpendicular bisector
  (c) Angle bisector (d) Circle
- **The angle bisectors of a triangle intersect at ___**
  (a) One point (b) Two points
  (c) Three points (d) Four points
- **Locus of all points equidistant from a fixed point is _____**
  (a) Circle
  (b) Perpendicular bisector
  (c) Angle bisector
  (d) Parallel lines
- **Locus of points equidistant from two fixed points is _____**
  (a) Circle
  (b) Perpendicular bisector
  (c) Angle bisector
  (d) Parallel lines
- **Locus of points equidistant from a fixed line is/are**
  (a) Circle
  (b) Perpendicular bisector
  (c) Angle bisector
  (d) Parallel lines
- **Locus of points equidistant from two intersecting lines is _____**
  (a) Circle
  (b) Perpendicular bisector
  (c) Angle bisector
  (d) Parallel lines
- **The set of all points which is farther than 2 km from a fixed point B is a region outside a circle of radius _____ and centre at B.**
  (a) 1 km (b) 1.9 km
  (c) 2 km (d) 2.1 km

Answer Key

1	b	**2**	a	**3**	c	**4**	a	**5**	a
6	a	**7**	b	**8**	d	**9**	c	**10**	c

- **Construct a right angled triangle with measures of sides 6 cm, 8 cm and 10 cm.**

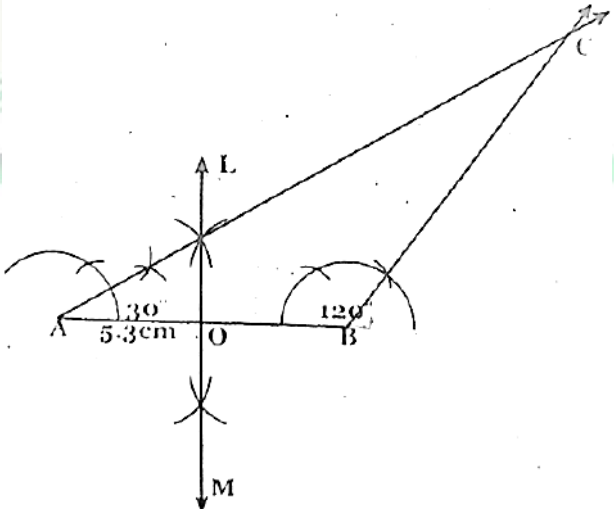
Ans:



12. Draw a line segment $m\overline{AB} = 6cm$
13. Take point B as centre, draw an arc of radius 8cm
14. Take point *A* as center, draw an arc of radius 10cm which cuts first arc to produce point *C*.
15. Join B and *A* to point *C*.
    So right angled triangle *ABC* is formed.

- **Construct a triangle** *ABC* **with** $m\overline{AB} = 5.3cm, m\angle A = 30°$ **and** $m\angle B = 120°$. **Draw the locus of all points which are equidistant from *A* and *B*.**

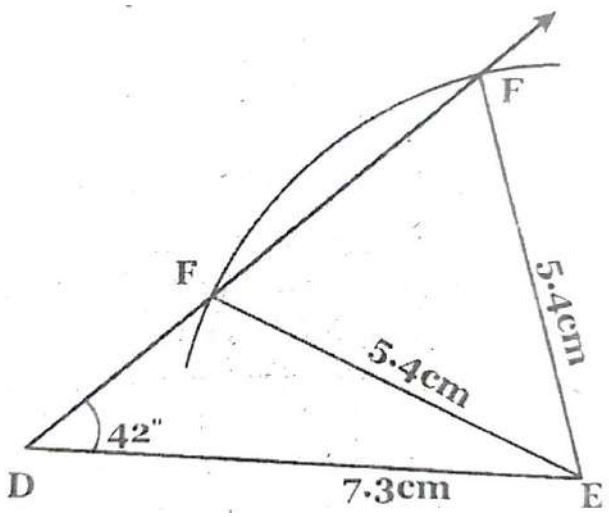
Ans:



- Draw a line segment $m\overline{AB} = 5.3cm$
- With the help of pair of compasses, construct $m\angle A = 30°$ and $m\angle b = 120°$
- Both angle arms intersect each other to produce point C. so ΔABC (obtuse) is formed.
- Draw perpendicular bisector of side $\overline{AB}$ to find locus of all points which are equidistant from *A* and *B*.

- **Construct a triangle with** $m\overline{DE} = 7.3cm, m\angle D = 42°$ **and** $m\overline{EF} = 5.4cm.$

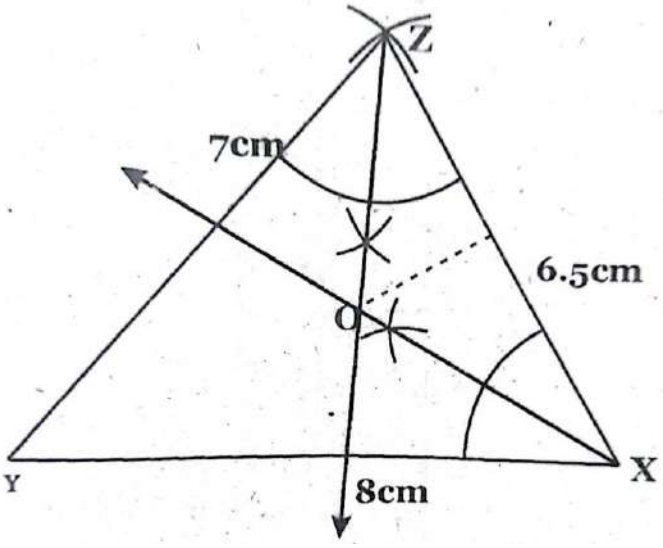
Ans:



- Draw a line segment $m\overline{DE} = 7.3cm$
- With the help of protractor draw an angle of 42° at point D.
- Take point E as centre, draw an arc of radius 5.4cm which cuts angle arm at two distinct points F' and F.
- Join E to F' and F.
  So two triangles are obtained.

- **Construct a triangle XYZ with $m\overline{YX} = 8cm, m\overline{YZ} = 7cm$ and $m\overline{XZ} = 6.5cm$. Draw the locus of all points which are equidistant from $\overline{XY}$ and $\overline{XZ}$.**

Ans:



- Draw a line segment $\overline{YX} = 8cm$
- Take point y as centre, mark an arc of radius 7cm.
- Take point X as centre mark another arc which intersect first arc to produce point Z.
- Join Y and X to Z to complete the triangle XYZ.
- Construct angle bisectors of $\angle X$ and $\angle Z$. The point of intersection of these angle bisectors give the locus of all points which are equidistant from $\overline{XY}$ and $\overline{XZ}$. Clearly, shown in diagram.

- **Construct a triangle FGH such that $m\overline{FG} = m\overline{GH} = 6.4cm, m\angle G = 122°$. Draw the locus of all**

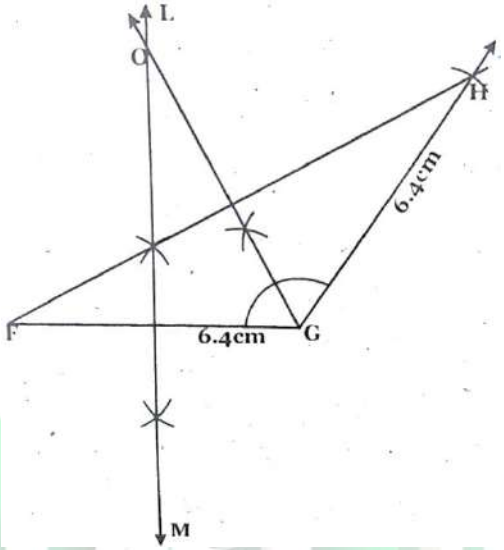
points which are:

16. Equidistant from F and G,

17. Equidistant from $\overline{FG}$ and $\overline{GH}$

18. **Mark the point where the two loci intersect.**

Ans:



19. With given measurements, construct triangle.
20. Find perpendicular bisector $\overleftrightarrow{LM}$ of side $\overline{FG}$. So locus of all points equidistant from points F and G lies on perpendicular bisector.
21. Find angle bisector of 122°

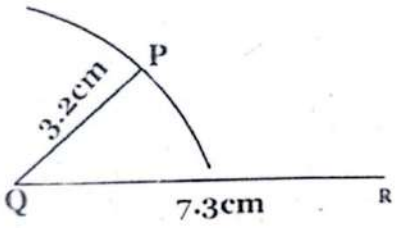
    So, locus of all points which are equidistant from $\overline{FG}$ and $\overline{GH}$ lies on angle bisector.
22. Two loci intersect each other at point *O*.

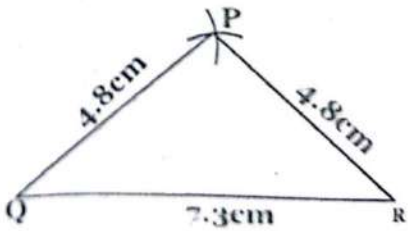
- **Two houses Q and R are 73 metres apart. Using a scale of 1 cm to represent 10m, construc the locus of a point P which moves such that it is:**

23. At a distance of 32 metres from Q

24. At a distance of 48 metres from the lines joining Q and R

Ans:





Here Q and R represent two houses. Distance between them is 73m (7.3 cm)

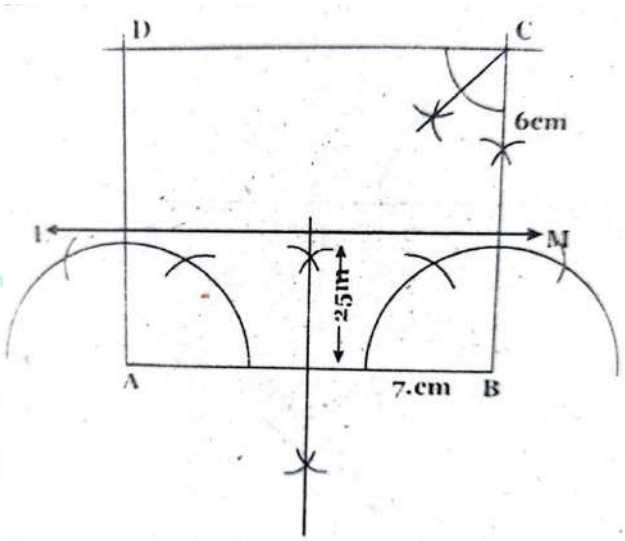
25. In first diagram.

Mark an arc of radius 3.2cm (32m) such that $\overline{QP} = 3.2cm(32m)$

26. In second diagram
    Take point Q as centre, mark an arc of radius 4.8 cm (48m)
27. Take R as centre, mark an arc of same radius, which cuts first arc to produce point P.
    Join Q to P & R to P

- **The field is in the form of a rectangle** *ABCD* **with** $m\overline{AB} = 70m$ **and** $m\overline{BC} = 60m.$ **Construct the rectangle** *ABCD* **using a scale of 1cm to represent 10m. Show that region inside the field which is less than 30 m from** *C* **and farther than 25 m from** $\overline{AB}$**.**

Ans:



28. Draw rectangle according to measurements.
29. Construct perpendicular bisector of $m\overline{AB}$
30. Draw a parallel line $\overleftrightarrow{LM}$ at a distance of 2.5cm from $\overline{AB}$
31. Find angle bisector of $\angle C$.

    Shade the region in the field which is less than 30m (3cm) from C and farther than 25m (2.5 cm) from $\overline{AB}$