

12 UNIT

INFORMATION HANDLING



Ungrouped and Grouped Data:

Data which is not arranged in any systematic order (groups or classes) is called ungrouped data. For example, the number of toys sold by a shopkeeper in a month is given below:

10, 5, 8, 12, 15, 20, 25, 30, 23, 15, 23, 21, 18, 15, 17, 23, 22, 15, 20, 21, 24, 18, 16, 21, 23, 21, 17, 19, 21, 23. This data is called ungrouped data.

If we arrange the above given data in groups or classes, then it is called grouped data.

Classes	Tally marks	No. of toys sold
5 – 9		2
10 – 14		2
15 – 19		10
20 – 24		14
25 – 29		1
30 – 34		1

Do you know?

Ungrouped data is also known as raw data.

Teachers' note!

By using more examples, clear the concept of grouped data and ungrouped data to the students.

In above grouped data, 5, 10, 15, 20, 25 and 30 are lower class limits and 9, 14, 19, 24, 29 and 34 are upper class limits.

Frequency Distribution:

A distribution or table that represents classes or groups along with their respective class frequencies is called frequency distribution. In other words, the various items of data are classified into certain groups or classes and the number of items lying in each group or class is put against that group or class. The data organised and summarized in this way is known as frequency distribution.

Think!

If the size of class limits is 6. The greatest value is 80 and the smallest value is 25. Can you find the number of class limits for the data?

Formation of Frequency distribution:

In this method, the raw data or the ungrouped data is presented into a grouped data. Choice is yours to select the number of classes.

Generally, the size of class limits is determined on the basis of the greatest value, smallest value and the desired number of groups or classes.

Following are the major steps to construct frequency distribution:

1. Find the range of the data. Range is the difference between the greatest value and the smallest value i.e., $\text{Range} = X_{\max} - X_{\min}$
2. Find the size of the class by dividing the range by the number of classes or groups you wish to make. For example, the greatest value is 136, the smallest value is 30 and if we have to make 10 classes or groups, then the size of class limits is found by the given formula.

$$\text{Size of Class} = \frac{\text{Range}}{\text{Number of classes}} = \frac{\text{Greater Value} - \text{Smallest Value}}{\text{Number of classes}}$$

$$= \frac{136 - 30}{10} = \frac{106}{10} = 10.6 \approx 11$$

So, size of class limits is 11

Keep in mind!

The number of times a value occurs in a data is called the frequency of that value. It is denoted by " f ".

Do you know?

Class boundaries may also be obtained from the midpoints (x) as $\left[x \pm \frac{h}{2} \right]$, where h is the difference between any two consecutive values

3. Prepare four columns.
 - (a) Class limits
 - (b) Tally marks
 - (c) Frequencies
 - (d) Class Boundaries
4. Make classes having size of 11. Start from the smallest value.
For example, 30-40, 41-51, 52-62 and so on.
5. Look for the class in which each element of ungrouped data falls. Draw a small tally mark (I) against that class and also tick the element concerned with a sign (✓). In this way you can remember that you have counted for the element. Continue this way with the next element that upto the last element of the data set. If 5 or more tallies appear in any class, mark every 5th tally diagonally as |||| .
6. Class boundaries usually are found by the following method:
 1. Chose the upper class limit of the 1st class and lower class limit of the 2nd class.
 2. Find the difference between these two limits.
 3. The difference is divided by 2 and subtract it from the lower class limit and add it to the upper class limit.

Example 1:

Following are the number of telephone calls made in a week to 30 teachers of a high school.

5 8 11 25 13 16 20 17 15 16 30 21 14 18 19
6 22 26 15 19 35 29 31 23 25 20 10 9 7 26

Construct a frequency distribution with number of classes 7.

Solution:

1. Find range
Greatest value (maximum value) = 35, Smallest value (minimum value) = 5
Range $X_{\max} - X_{\min} = 35 - 5 = 30$
2. Size of class limits $= \frac{\text{Range}}{\text{Number of classes}} = \frac{30}{7} = 4.28 \approx 5$
3. Make class limits having size 5. For example, 5-9, 10-14, 15-19 and so on.
(see 1st column of table: 1).
4. Tally marks are used to count the values, fall in the given class limits.
(See 2nd column of table: 1).
5. Now, count the number of tally marks and write the number as frequency in the third column (see 3rd column of table: 1).
6. **Class boundaries**
The difference between lower class limit of the second class and upper class limit of the first class is 1. i.e., $10 - 9 = 1$. Now, divide the difference of the limits by 2 i.e., $\frac{1}{2} = 0.5$.

Activity

Collect data of height of 50 students in your class, and convert the data into grouped data.

Lower class boundaries are obtained by “subtracting 0.5” from the lower class limits.

Upper class boundaries are obtained by “adding 0.5” to the upper class limits.

Lower class boundaries

5 – 0.5

4.5 and so on.

(see 4th column of the table: 1)

Upper class boundaries

9+0.5

9.5 and so on.

Table 1

Class limits	Tally marks	Frequency (f)	Class Boundaries (C.B)
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5 – 9		5	4.5 – 9.5
10 – 14		4	9.5 – 14.5
15 – 19		8	14.5 – 19.5
20 – 24		5	19.5 – 24.5
25 – 29		5	24.5 – 29.5
30 – 34		2	29.5 – 34.5
35 – 39		1	34.5 – 39.5

Graph of Frequency Distribution:

The following are the types of graphs which can be used to represent a frequency distribution on a graph.

(a) Histogram

(b) Frequency polygon

(a) Histogram (with equal class limits)

This is a graph of adjacent rectangles constructed on xy plane. A histogram is similar to bar graph but it is constructed for a frequency distribution. In a histogram, the values of the data (classes) are represented along the horizontal axis and the frequencies are shown by bars perpendicular to the horizontal axis. Bars of equal width are used to represent individual classes of frequency table. The procedure for making histogram is explained below:

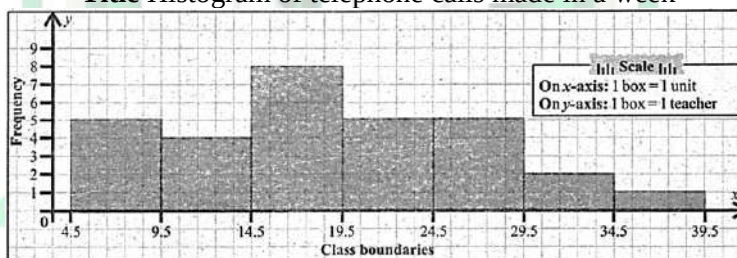
Do you know?

Continuous data is mostly represented by using histogram and frequency polygon.

1. Draw lines as x -axis and as y -axis on a graph paper perpendicular to each other.
2. Class boundaries are marked on x -axis and a rectangle is made against each group with its width proportional to the size of the class limits and height proportional to the class frequencies.
3. Setting a scale, draw frequencies on y axis. The resulting figure is called histogram.

Histogram of table: 1 is given below:

Title Histogram of telephone calls made in a week

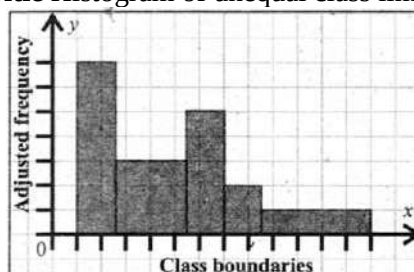


Histogram (with unequal class limits)

The procedure for making histogram is explained below:

1. Draw lines as x -axis and y -axis on a graph paper perpendicular to each other.
2. Class boundaries are marked on x -axis and a rectangle is made against each group with its width proportional to the size of class limits and height proportional to the class frequencies.
3. This can be achieved by adjusting the heights of rectangle. The height of each rectangle is obtained by dividing each class frequency on its class limit size.

Title Histogram of unequal class limits



EXERCISE 12.1

1. The following distribution represents the scores achieved by a group of chemistry students

in the chemistry laboratory.

Scores	24 – 28	29 – 33	34 – 38	39 – 43	44 – 48	49 – 53	Total
No. of students	3	6	12	23	15	6	65

Answer the following questions

1. What is the upper limit of the last class?

Ans:

53

2. What is the lower limit of the class 39-43?

Ans:

39

3. What is the midpoint of the class (34 – 38)?

Ans:

36

4. What are the class frequencies of the classes 29 – 33 and 44 – 48?

Ans:

6 and 15

5. What is the size of the class limits in the above frequency distribution?

Ans:

5

6. In which class or group does minimum number of students fall?

Ans:

24 – 28

7. What is the lower limit of the class having 15 as its class frequency?

Ans:

44

8. What is the number of students having scores between 24 and 43?

Ans:

44

9. For a school staff, the following expenditures (rupees in hundred) are required for the repair of chairs.

145, 152, 153, 156, 158, 160, 146, 152, 155, 159,
161, 163, 165, 147, 148, 151, 154, 156, 158, 160,
144, 167, 151, 150, 152, 149, 145, 153, 152, 155

Prepare a frequency distribution by tally bar method using 3 as the size of class limits and also write down what are the frequencies of the last three classes?

Ans:

Class limit	144 – 146	147 – 149	150 – 152	153 – 155	156 – 158	159 – 161	162 – 164	165 – 167
Tally marks	IIII	III	IIII II	IIII	IIII	IIII	I	II
f	4	3	7	5	4	4	1	2

$\sum f = 30$

10. Given below are the weights in kg of 30 students of a high school.

30, 33, 24, 21, 15, 39, 37, 44, 42, 33,
33, 28, 29, 32, 31, 28, 26, 32, 34, 35,
38, 36, 41, 30, 35, 41, 23, 26, 18, 34

Taking 5 as the size of the class limit, prepare a frequency table and construct a frequency polygon.

Ans:

Class limit	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44
Tally marks	II	IIII	IIII	IIII IIII	IIII I	IIII
f	2	3	5	10	6	4

$\sum f = 30$

11. A group of Grade –10 students obtained the following marks out of 100 marks in English test

58, 59, 58, 33, 40, 58, 45, 46, 43, 45, 45,
50, 52, 49, 50, 57, 52, 55, 49, 50, 62, 49,
48, 44, 42, 47, 46, 47, 46, 53, 40, 44

Classify the data into a frequency distribution by (direct method) taking 6 as the size of class limit. Also find the class limit with least class frequency and construct histogram for the data.

Ans:

Class marks	33–38	39–44	45–50	51–56	57–62
Tally marks	I	I			I
f	1	6	15	4	6

$\Sigma f = 30$

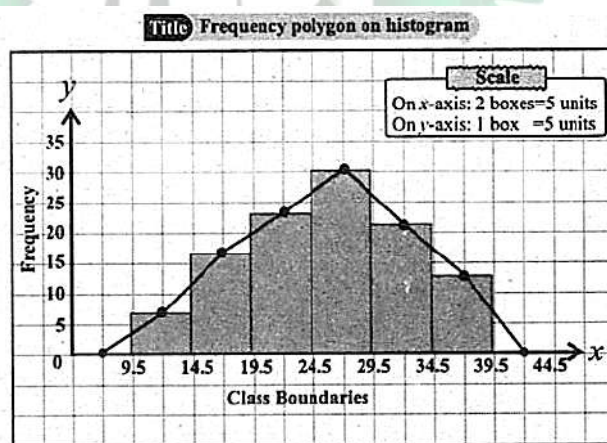
12. From the table given below. Draw a frequency polygon on histogram for the given frequency distribution.

Weight (kg)	10–14	15–19	20–24	25–29	30–34	35–39
Frequency (f)	06	17	23	30	22	13

Ans:

Weight (kg)	10–14	15–19	20–24	25–29	30–34	35–39
Frequency (f)	6	17	23	30	22	13
Class boundaries	9.5–14.5	14.5–19.5	19.5–24.5	24.5–29.5	29.5–34.5	34.5–39.5

Frequency polygon on histogram



13. The following data shows the number of heads in an experiment of 50 sets of tossing a coin 5 times. Make a discrete frequency distribution from the information.

3, 3, 4, 0, 5, 4, 3, 3, 1, 2, 4, 5, 0, 3, 2, 4, 4, 0, 0, 0, 5, 5, 3, 2, 1,
4, 3, 2, 5, 3, 2, 1, 3, 5, 4, 3, 2, 1, 3, 2, 1, 3, 1, 3, 1, 4, 3, 2, 2, 4

Ans:

No. of heads	0	1	2	3	4	5
Tally marks		II				I
f	5	7	9	14	9	6

$\sum f = 50$

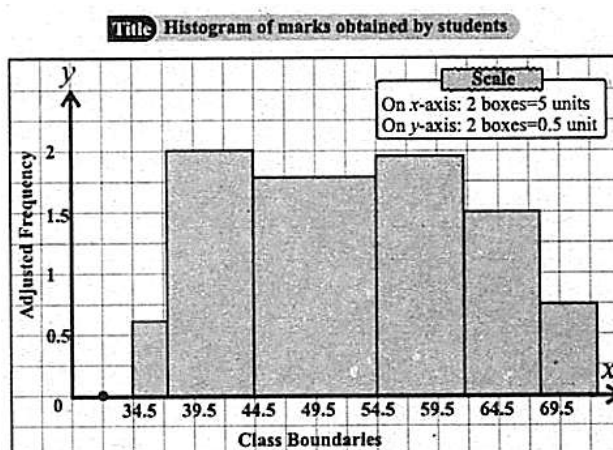
14. The marks obtained by the students of Grade –10 in mathematics test were grouped into the following frequency distribution.

Marks	35 – 37	38 – 44	45 – 54	55 – 61	62 – 67	68 – 72
Frequency	2	12	16	13	9	3

Draw a histogram for the above distribution

Ans:

Marks	35 – 37	38 – 44	45 – 54	55 – 61	62 – 67	68 – 72
Frequency	2	12	16	13	9	3
	34.5 – 37.5	37.5 – 44.5	44.5 – 54.5	54.5 – 61.5	61.5 – 67.5	67.5 – 72.5

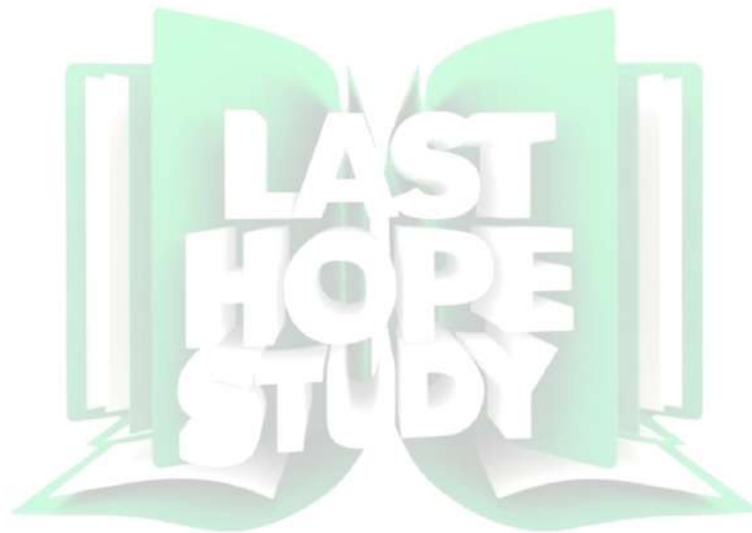
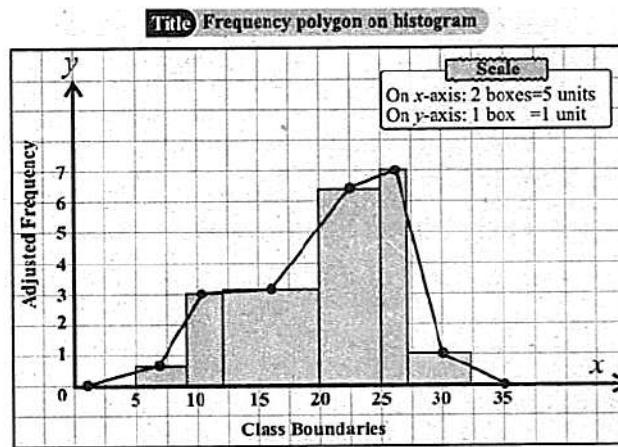


15. Make a frequency polygon on histogram for the following grouped data:

Class limits	5 – 8	8 – 12	12 – 20	20 – 25	25 – 27	27 – 32
Frequency (f)	2	12	25	32	14	5

Ans:

Class limits	5 – 8	8 – 12	12 – 20	20 – 25	25 – 27	27 – 32
Frequency (f)	2	12	25	32	14	5
Mid point	$\frac{5+8}{2} = 6.5$	$\frac{8+12}{2} = 10$	$\frac{12+20}{2} = 16$	$\frac{20+25}{2} = 22.5$	$\frac{25+27}{2} = 26$	$\frac{27+32}{2} = 29.5$



Measures of Location (Central Tendency):

The measure that gives the centre of the data is called measure of central tendency. Therefore, measure of central tendency is used to find out the middle or central value of a data set.

We have seen that when the raw data has been condensed into a frequency distribution, the information was easily understood. The information given in the data can be further condensed to a single representative value for the entire distribution. It is more or less the central value around which the data appear to be crowded. For example, usually, we make statements such as:

1. Hassan studies 6 hours daily.
2. The monthly expenditure of Ayesha's house is Rs.50,000.
3. The speed of Maham's car is 72 km per hour.
4. In a country, yearly income is 70,000 rupees per head.
5. The price of onion in the market is Rs.150 per kg etc.

If we look at the first statement, we come to know that Hassan does not study exactly 6 hours daily. Sometimes, he studies more than 6 hours and sometimes less. But still why do we say that he studies 6 hours daily? As he studies near about 6 hours daily so in his study time, 6 hours becomes an important figure because of its approximated statement, which we call Average. Such an average value is known a measure of central tendency because it is a representative value of the daily study time. Similarly, other statements can also be treated as representative values. As each statement locates the centre of a distribution so it is also known as a measure of central tendency.

The following measure of central tendency will be discussed in this section:

- | | |
|----------------------------|--------------------|
| (i) Arithmetic Mean (A.M.) | (ii) Median |
| (iii) Mode | (iv) Weighted mean |

Arithmetic Mean (A.M.)

It is defined as a value of variable which is obtained by dividing the sum of all the values (observations) by their number of observations. Thus, the arithmetic mean of a set of values $x_1, x_2, x_3, \dots, x_n$ is denoted by $\bar{X}$ (read as X -bar) and is calculated as:

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\sum x}{n} \text{ (Direct method)}$$

where, the sign $\sum$ stands for the sum and n is the number of observations.

Example 4:

The marks of a student in five examinations were 64, 75, 81, 87, 90. Find the arithmetic mean of the marks.

Solution:

$$\begin{aligned} A.M. = \bar{X} &= \frac{\sum x}{n} \\ &= \frac{64 + 75 + 81 + 87 + 90}{5} \end{aligned}$$

$$\text{Or } \bar{X} = \frac{397}{5} = 79.4 \text{ marks}$$

Method of finding Arithmetic Mean for Grouped Data

Let $x_1, x_2, x_3, \dots, x_n$ be the midpoints of the class limits with corresponding frequencies say $f_1, f_2, f_3, \dots, f_n$. Then the arithmetic mean is obtained by dividing sum of the products of f and x by the sum of all the frequencies.

$$\bar{X} = \frac{f_1x_1 + f_2x_2 + \dots + f_nx_n}{f_1 + f_2 + \dots + f_n} = \frac{\sum fx}{\sum f}$$

Try Yourself!

The mean of 10, 30, 40, x , 67 and 81 is 50. Find the value of the x .

Example 6:

Given below are the marks out of 100 obtained by 100 students in a examination. Find the average marks of the students.

Marks	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60
No. of students	14	16	18	23	18	11

Solution:

Marks	Midpoint (x)	Frequency (f)	fx
30 – 35	32.5	14	455.0
35 – 40	37.5	16	600.0
40 – 45	42.5	18	765.0
45 – 50	47.5	23	1092.5
50 – 55	52.5	18	945.0
55 – 60	57.5	11	632.5

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{4490}{100}$$

$$\bar{X} = 44.9 \text{ marks}$$

Hence, the average marks is 44.9 of the students.

Short Formula for Computing Arithmetic Mean

The computation of arithmetic mean using direct method for ungrouped data as well as for grouped data is no doubt easy for small values. If x and f become very large, it becomes difficult to deal with the problems so to minimize our time and calculations we take deviations from an assumed or provisional mean. Let A be considered as assumed or provisional mean (may be any value from the values of x or any number) and D denotes the deviations of x from A i.e., $D = x - A$. for $x = D + A$, the formula of arithmetic mean becomes:

$$\bar{X} = A + \frac{\sum D}{n} \quad (\text{for ungrouped data}) \quad \dots(i)$$

$$\bar{X} = A + \frac{\sum fD}{\sum f} \quad (\text{for grouped data}) \quad \dots(ii)$$

Example 7:

Find the arithmetic mean using short formula for the runs made by a batsman.

Runs: 40, 45, 50, 52, 50, 60, 56, 70

Solution:

Taking deviations from $A = 52$ (assumed mean)

x	40	45	50	52	50	60	56	70
D = x - A	-12	-7	-2	0	-2	8	4	18

$$\text{Now: } \sum D = -23 + 30 = 7$$

$$\therefore \bar{X} = A + \frac{\sum D}{n}$$

$$\text{So, } \bar{X} = 52 + \frac{7}{8}$$

$$= 52 + 0.875 = 52.875 \text{ or } 53 \text{ runs.}$$

Try Yourself!

If $\bar{X} = 120$; $A = 85$ and $n = 25$, then can you find the value of $\sum D$?

Example 8:

Deviations from 12.5 of ten different values are 6, -2, 3.5, 9, 8.7, -5.5, 14, 11.3, -6.8, 4.2, find the arithmetic mean.

Solution:

Deviations from 12.5 are: 6, 2, 3.5, 9, 8.7, 5.5, 14, 11.3, -6.8, 4.2

Now, $\sum D = 34$ Also, $A = 12.5$ using the formula we have.

$$\bar{X} = A + \frac{\sum D}{n}$$

$$= 12.5 + \frac{34}{10}$$

$$\text{or } \bar{X} = 12.5 + 3.4 = 15.9$$

Example 9:

The heights (in inches) of 200 students are recorded in the following frequency distribution.

Find the mean height of the student by short formula.

Height (x) (in inches)	51	52	53	54	55	56	57	58	59	60
Frequency(f)	2	5	8	24	55	45	38	16	6	1

Solution:

Heights (x) (in inches)	Frequency (f)	$A = 55$ $D = x - A$	fD
51	2	-4	-8
52	5	-3	-15
53	8	-2	-16
54	24	-1	-24
$A \leftarrow 55$	55	0	0
56	45	1	45
57	38	2	76
58	16	3	48
59	6	4	24
60	1	5	5
Total	$\sum f = 200$		$\sum fD = 135$

Now, using the formula (ii). we get

$$\bar{X} = A + \frac{\sum fD}{\sum f}$$

$$\bar{X} = 55 + \frac{135}{200}$$

$$\text{or } \bar{X} = 55 + 0.675$$

$$\therefore \bar{X} = 55.68 \text{ inches approx.}$$

Example 10:

Ten students each from Grade-V section A and B of a well reputed school were taken

randomly. Their weights were measured in kg. and recorded as given below:

Weights (kg) Section A	30	28	32	29.5	35	34	31	33	40	37.5
Weights (kg) Section B	35	31.5	34.5	35	32.8	38	29.5	36	36.5	34

1. Compute the mean weight for section A and B.
2. Conclude which section is better on Average?

Solution:

1. We find arithmetic mean for both the sections by direct method. (Any method can be applied).
As number of observations $n = 10$

$$\text{and } \bar{X}_{(A)} = \frac{\sum X_A}{n}$$

$$\therefore \bar{X}_{(A)} = \frac{330}{10} = 33\text{kg}$$

$$\text{and } \bar{X}_{(B)} = \frac{\sum X_{(B)}}{n}$$

$$\therefore \bar{X}_{(B)} = \frac{342.8}{10} = 34.28\text{kg}$$

$X_{(A)}$	$X_{(B)}$
30	35
28	31.5
32	34.5
29.5	35
35	32.8
34	38
31	29.5
33	36
40	36.5
37.5	34
$\sum X_{(A)} = 330$	$\sum X_{(B)} = 342.8$

2. We have seen from the results that $\bar{X}_{(B)}$ is greater than $\bar{X}_{(A)}$. Therefore, we conclude that section B is better on the average.

Median

Median is the middle most value in an arranged (ascending or descending order) data set. Median is the value which divides the data into two equal parts i.e., 50% data is before the median and 50% data after it. Median is denoted by $\tilde{X}$.

Median for ungrouped data

The median of n observations $x_1, x_2, \dots, x_n$ is obtained as:

$$\text{Median } (\tilde{X}) = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ observation (when } n \text{ is odd number)}$$

$$\text{Median } (\tilde{X}) = \frac{1}{2} \left(\left(\frac{n}{2} \right)^{\text{th}} \text{ observation} + \left(\frac{n+1}{2} \right)^{\text{th}} \text{ observation} \right) \text{ (when } n \text{ is even number)}$$

Example 11:

The following are the scores made by a batsman. Find the median of the data. 8, 12, 18, 13, 16, 5, 20.

Solution:

Writing the scores in an ascending order, we have 5, 8, 12, 13, 16, 18, 20

Since, number of observations is odd i.e., $n = 7$

$$\begin{aligned}\text{Median } (\tilde{X}) &= \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation} \\ &= \left(\frac{7+1}{2}\right)^{\text{th}} \text{ observation} = 4^{\text{th}} \text{ observation} = 13\end{aligned}$$

Hence, 13 is the median of the given data.

Example 12:

Following are the marks out of 100 obtained by 10 students in English. 23, 15, 35, 48, 41, 5, 8, 9, 11, 51. Find the median of the data.

Solution:

Arranging the data in an ascending order. 5, 8, 9, 11, 15, 23, 35, 41, 48, 51

Since, number of observation is even, i.e., $n = 10$

$$\therefore \text{Median } (\tilde{X}) = \frac{1}{2} \left(\left(\frac{n}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{n+2}{2}\right)^{\text{th}} \text{ observation} \right)$$

$$\text{As, } \frac{n}{2} = \frac{10}{2} = 5 \text{ and } \frac{n+2}{2} = \frac{12}{2} = 6$$

$$\therefore \text{Median } = \frac{1}{2} [5^{\text{th}} \text{ observation} + 6^{\text{th}} \text{ observation}]$$

$$\text{Or Median} = \frac{1}{2} [15 + 23] = \frac{38}{2} = 19$$

Hence, 19 is the median of the data.

Median for Grouped Data

The median for grouped data is obtained by the following formula:

$$\text{Median } (\tilde{X}) = \ell + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

Where, ℓ = Lower class boundary of median class,

h = The size of class limits of median class,

f = Frequency of the median class,

n = Total frequency i.e., $\sum f$,

and c = Cumulative frequency preceding the median class.

Remember the following points:

Q.1 The groups of classes must be in a continuous form i.e., we need class boundaries.

Q.2 Make the column of cumulative frequencies ($c.f.$) from the column of frequencies.

Q.3 Locate median class i.e., $\left(\frac{n}{2}\right)^{\text{th}}$ see value in $c.f.$ column wherever it lies.

Q.4 Underline the median class, then take the values of f and h of the median class thus obtained.

Example 13:

The heights of 100 athletes, measured to the nearest (inches) are given in the following table.
Find the median.

Heights (in inches)	62.5–63.5	63.5–64.5	64.5–65.5	65.5–66.5	66.5–67.5	67.5–68.5	68.5–69.5	69.5–70.5	70.5–71.5
No. of Students	4	6	10	20	30	13	12	3	2

Solution:

In the above data, class boundaries have already been given:

Heights (inches)	Frequency (f)	c.f.
62.5 – 63.5	4	4
63.5 – 64.5	6	6 + 4 = 10
64.5 – 65.5	10	10 + 10 = 20
65.5 – 66.5	20	20 + 20 = 40 → c
66.5 – 67.5	30	30 + 40 = 70 →
67.5 – 68.5	13	13 + 70 = 83
68.5 – 69.5	12	12 + 83 = 95
69.5 – 70.5	3	3 + 95 = 98
70.5 – 71.5	2	2 + 98 = 100 → n
Total	Σf = 100	---

Here, $n = 100$

$$\text{So, } \frac{n}{2} = \frac{100}{2} = 50$$

50th item lies in the class boundaries 66.5 – 67.5.

$$\ell = 66.5, h = 1, f = 30, c = 40$$

$$\begin{aligned} \therefore \text{Median} &= \ell + \frac{h}{f} \left(\frac{n}{2} - c \right) = 66.5 + \frac{1}{30} (50 - 40) \quad (\text{Putting the values}) \\ &= 66.5 + \frac{10}{30} = 66.5 + 0.33 \end{aligned}$$

$$\therefore \text{Median} = 66.83 \text{ inches}$$

Example 14:

Following are the weights (in kg) of 50 men. Find the median weight.

Weights (kg)	110 – 114	115 – 119	120 – 124	125 – 129	130 – 134
No. of men	5	12	23	6	4

Solution:

As class boundaries are not given so, first of all we make class boundaries by the usual procedure.

Weight (kg)	Frequency (f)	Class Boundaries	c.f.
110 – 114	5	109.5 – 114.5	5
115 – 119	12	114.5 – 119.5	17 → c
120 – 124	23	119.5 – 124.5	40 →
125 – 129	6	124.5 – 129.5	46
130 – 134	4	129.5 – 134.5	50 → n
Total	Σf = 100	---	---

Here $n = 50$, so, $\frac{n}{2} = \frac{50}{2} = 25$. 25th item lies in 119.5 – 124.5.

$$\ell = 119.5, h = 5, f = 23, c = 17$$

$$\begin{aligned} \text{Median} &= \ell + \frac{h}{f} \left(\frac{n}{2} - c \right) = 119.5 + \frac{5}{23} (25 - 17) \quad (\text{Putting the values}) \\ &= 119.5 + \frac{40}{23} = 119.5 + 1.74 \end{aligned}$$

$$\therefore \text{Median} = 121.24 \text{ kg}$$

Mode:

In a data the values (observation) which appears or occurs most often is called mode of the data.

It is the most common value. Mode is denoted by $\tilde{X}$.

Mode for Ungrouped Data:

Example 15.

The marks in mathematics of Jamal in eight monthly tests were 75, 76, 80, 80, 82, 82, 82, 85.

Find the mode of the marks.

Solution:

As 82 is repeated more than any other number so, clearly mode is 82.

Example 16:

Ten students were asked about the number of questions they have solved out of 20 questions last week. Records were 13, 14, 15, 11, 16, 10, 19, 20, 18, 17. Find the mode of the data.

Solution:

It is obvious that the given data contains no mode. It is ill-defined. Sometimes data contains several modes. If the data is: 10, 15, 15, 15, 20, 20, 20, 25, 32, then data contains two modes i.e., 15 and 20.

Example 17:

A survey was conducted from the 15 students of a school and asked the students about their favourite colour.

The responses are: purple, yellow, purple, yellow, yellow, red, blue, green, yellow, yellow, red, blue, yellow, purple, green. Find mode of the data.

Solution:

Mode is the most frequent colour.

Mode = yellow So, the colour “yellow” is the mode of the given data.

Mode for Grouped Data

Mode can be calculated by the following formula:

$$\text{Mode} = \ell + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h$$

Where, ℓ = Lower class boundary of the modal class.

f = Frequency of the modal class.

h = Size of the modal class.

f_1 = Frequency preceding the modal class.

f_2 = Frequency following the modal class and

Remember!

A data can has more than one mode.
A data may or may not have a mode.

Note:

Mode cannot be easily calculated from the data presented in a frequency distribution. As it has no individual values, so we do not know which value appears most frequently. We only assume the class with the highest frequency as a modal class.

Example 18:

Following are the heights in (inches) of 40 students in Grade – 8.

Heights (inches)	48 – 50	50 – 52	52 – 54	54 – 56	56 – 58	58 – 60
No. of students	5	7	10	9	6	3

Find mode of the above data.

Solution:

Heights (inches)	Frequency (ℓ)
48 – 50	5
50 – 52	$7 \rightarrow f_1$
52 – 54	$10 \rightarrow f_m$
54 – 56	$9 \rightarrow f_2$
56 – 58	6
58 – 60	3
Total	$\sum f = 40$

In the above data, class boundaries have already been given. Using the formula for grouped data we find mode as:

$$\ell = 52, h = 2, f_m = 10, f_1 = 7, f_2 = 9$$

$$\text{Mode} = \ell + \frac{(f_m - f_1) \times h}{(f_m - f_1) + (f_m - f_2)}$$

Or $\text{Mode} = 52 + \frac{(10 - 7) \times 2}{(10 - 7) + (10 - 9)}$

Or $\text{Mode} = 52 + \frac{3 \times 2}{3 + 1} = 52 + \frac{6}{4}$

Or $\text{Mode} = 52 + 1.5 = 53.5$ (inches)

Weighted Mean:

Arithmetic mean is used when all the observations are given equal importance / weight but there are certain situations in which the different observations get different weights. In this situation, weighted mean denoted by $\bar{X}_w$ is preferred. The weighted mean of $X_1, X_2, X_3, \dots, X_n$ with corresponding weights $W_1, W_2, W_3, \dots, W_n$ is calculated as:

$$\bar{X}_w = \frac{W_1 X_1 + W_2 X_2 + W_3 X_3 + \dots + W_n X_n}{W_1 + W_2 + W_3 + \dots + W_n} = \frac{\sum_{i=1}^n W_i X_i}{\sum_{i=1}^n W_i} = \frac{\sum WX}{\sum W}$$

Example 19:

The following data describes the marks of a student in different subjects and weights assigned to these subjects are also given:

Mark (X)	74	78	74	90
Weights (W)	4	3	5	6

Find its weighted mean.

Solution:

$$\begin{aligned} \text{Weighted mean } (\bar{X}_w) &= \frac{\sum WX}{\sum W} \\ \bar{X}_w &= \frac{4(74) + 3(78) + 5(74) + 6(90)}{4 + 3 + 5 + 6} \\ \bar{X}_w &= \frac{296 + 234 + 370 + 540}{18} = \frac{1440}{18} \\ \bar{X}_w &= 80 \end{aligned}$$

Activity!

Collect data of weights of 50 students. Make a frequency distribution and find mean, median and mode of the data.

Skill practice!

Find the mean, median and mode of the first twenty whole numbers.

EXERCISE 12.2

1. Find the arithmetic mean in each of the following

1. 4, 6, 10, 12, 15, 20, 25, 28, 30

Ans:

x	4	6	10	12	15	20	25	28	30	$\sum x = 150$
---	---	---	----	----	----	----	----	----	----	----------------

$$\bar{n} = \frac{\sum x}{n}$$

$$\sum x = 150 \quad n = 9$$

$$\bar{x} = \frac{150}{9} \Rightarrow \bar{x} = 16.67$$

2. 12, 18, 19, 0, -19, -18, -12

Ans:

x	12	18	19	0	-19	-18	-12	$\sum x = 0$
---	----	----	----	---	-----	-----	-----	--------------

$$\sum x = 0 \quad n = 7$$

$$\bar{x} = \frac{0}{7} \Rightarrow \bar{x} = 0$$

3. 6.5, 11, 12.3, 9, 8.1, 16, 18, 20.5, 25

Ans:

x	6.5	11	12.3	9	8.1	16	18	20.5	25	$\sum x = 126.4$
---	-----	----	------	---	-----	----	----	------	----	------------------

$$\sum x = 126.4 \quad n = 9$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{126.4}{9} \Rightarrow \bar{x} = 14.04$$

4. 8, 10, 12, 14, 16, 20, 22

Ans:

x	8	10	12	14	16	20	22	$\sum x = 102$
---	---	----	----	----	----	----	----	----------------

$$\sum x = 102 \quad n = 7$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{102}{7} \Rightarrow \bar{x} = 14.57$$

5. Following are the heights in (inches) of 12 students. Find the median height.

55, 53, 54, 58, 60, 61, 62, 56, 57, 52, 51, 63

Ans:

x	51	52	53	54	55	56	57	58	60	61	62	63
---	----	----	----	----	----	----	----	----	----	----	----	----

$$\text{Median} = \tilde{n} = \frac{1}{2} \left[\frac{n+1}{2} \text{ observation} + \left(\frac{n+2}{2} \right) \text{ observation} \right]$$

$$= \frac{1}{2} \left[6 \left(\frac{12+2}{2} \right) \right] = \frac{1}{2} [6 + 17^{\text{th}}] = \frac{1}{2} [56 + 57] = \frac{1}{2} [113]$$

Median height = 56.5 inches

6. Following are the earnings (in Rs.) of ten workers:

88, 70, 72, 125, 115, 95, 81, 90, 95, 90. Calculate

Ans:

x	70	72	81	88	90	90	95	95	115	125	$\sum x = 921$
---	----	----	----	----	----	----	----	----	-----	-----	----------------

- **Arithmetic Mean**

Ans:

$$\bar{x} = \frac{\sum x}{n}$$

$$\sum x = 921 \quad n = 10$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{921}{10} \Rightarrow \bar{x} = 92.1$$

- **Median**

Ans:

$$\text{Median} = \tilde{n} = \frac{1}{2} \left[\frac{n^{th}}{2} \text{ observation} + \left(\frac{n+2}{2} \right)^{th} \text{ observation} \right]$$

$$n = 10$$

$$\tilde{n} = \frac{1}{2} \left[\left(\frac{10}{2} \right)^{th} + \left(\frac{10+2}{2} \right)^{th} \right]$$

$$\tilde{n} = \frac{1}{2} [5^{th} + 6^{th}]$$

$$\tilde{n} = \frac{1}{2} [90 + 90]$$

$$\tilde{n} = \frac{1}{2} (180)$$

$$\tilde{n} = 90$$

- **Mode**

Ans:

Mode = the most repeated value = 90 and 95

7. **The Marks obtained by the students in the subject of English are given below**

Mark obtained	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39
Frequency	9	18	35	17	5

Find:

Ans:

Marks obtained	Frequency	x	fx	$D = x - A$	fD
15–19	9	17	153	17–17=0	0
20–24	18	22	396	22–17=5	90
25–29	35	27	945	27–17=10	350
30–34	17	32	544	32–17=15	255
35–39	5	37	185	37–17=20	100
	$\sum f = 84$		$\sum fx = 2223$		$\sum fD = 795$

1. **Arithmetic mean of their marks by direct and short formula.**

Ans:

Direct method

$$\sum fx = 2223 \quad \sum f = 84$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$\bar{x} = \frac{2223}{84}$$

$$\bar{x} = 26.46$$

$$\bar{x} = A + \frac{\sum fD}{\sum f}$$

$$A = 17 \quad \sum f = 84$$

$$\sum fD = 795$$

$$\bar{x} = 17 + \frac{795}{84}$$

$$\bar{x} = 17 + 9.46$$

$$\bar{x} = 26.46$$

2. **Median of their marks.**

Ans:

Marks	Class Boundaries	f	c.f
15-19	14.5-19.5	9	9
20-24	19.5-24.5	18	27 (c)
25-29	(l) 24.5-29.5	35 (f)	62
30-34	29.5-34.5	17	79
35-39	34.5-39.5	5	84

Here $n = 84$

$$\frac{n}{2} = \frac{84}{2} = 42$$

42nd item lies in the class boundaries

24.5 - 29.5

$$\text{Using Median} = \ell + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

$$= 24.5 + \frac{5}{35} \left(\frac{84}{2} - 27 \right)$$

$$= 24.5 + \frac{1}{7} (42 - 27)$$

$$= 24.5 + \frac{15}{7}$$

$$24.5 + 2.143$$

$$\tilde{x} = 26.64$$

3. **Given below is a frequency distribution.**

Class Interval	5-9	10-14	15-19	20-24	25-29
Frequency	1	8	18	11	2

Find the mode of the frequency distribution.

Ans:

Class Interval	5-9	10-14	15-19	20-24	25-29
Frequency	1	$8f_1$	$18f_m$	$11f_2$	2
Class boundaries	4.5-9.5	9.5-14.5	14.5-19.5	19.5-24.5	24.5-29.5

$$\text{Mode} = \hat{x} = \ell + \frac{(f_m - f_1)h}{(f_m - f_1) + (f_m - f_2)}$$

$$\ell = 14.5 \quad f_m = 18 \quad f_1 = 8 \quad f_2 = 11 \quad h = 5$$

$$\hat{x} = 14.5 + \frac{(18-8)5}{(18-8) + (18-11)}$$

$$= 14.5 + \frac{10 \times 5}{10 + 7} = 14.5 + \frac{50}{17}$$

$$= 14.5 + 2.94 = 17.44$$

4. **Ten boys work on petrol pump station. They get weekly wages as follows:**

Wages (in Rs.) 4250, 4350, 4400, 4250, 4350, 4410, 4500, 4500, 4390. Find the arithmetic mean by short formula, median and mode of their wages.

Ans:

x	$D = x - A$
42050	$4250 - 4250 = 0$
4250	$4300 - 4250 = 0$
4300	$4300 - 4250 = 50$
4350	$4350 - 4250 = 100$
4350	$4350 - 4250 = 100$
4390	$4390 - 4250 = 140$
4400	$4400 - 4250 = 150$
4410	$4400 - 4250 = 160$
4500	$4500 - 4250 = 250$
4500	$4500 - 4250 = 250$
$\Sigma = 43700$	$\Sigma D = 1200$

Mode = the most frequent value
= 4250, 4350, 4500

$$\bar{x} = A + \frac{\Sigma D}{n}$$

$$\bar{x} = 4250 + \frac{1200}{10}$$

$$\bar{x} = 4250 + 120$$

$$\bar{x} = 4370$$

Median

$$\text{Median} = \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} \text{observation} + \left(\frac{n+2}{2} \right)^{\text{th}} \text{observation} \right]$$

$$\text{Median} = \tilde{x} = \frac{1}{2} \left[\left(\frac{10}{2} \right)^{\text{th}} + \frac{10+2}{2} \right]$$

$$\text{Median} = \tilde{x} = \frac{1}{2} [5^{\text{th}} + 6^{\text{th}}]$$

$$\text{Median} = \tilde{x} = \frac{1}{2} [4350 + 4390]$$

$$\tilde{x} = \frac{1}{2} [8740]$$

$$\tilde{x} = 4370$$

5. **The arithmetic mean of 45 numbers is 80. Find their sum.**

Ans:

$$n = 45 \quad \bar{x} = 80 \quad \Sigma x = ?$$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\Sigma x = n\bar{x}$$

$$\Sigma x = 45 \times 80$$

$$\Sigma x = 3600$$

6. **Five number are 1, 4, 0, 7, 9. Find their mean, median and mode.**

Ans:

x	0	1	4	7	9	$\Sigma = 21$
-----	---	---	---	---	---	---------------

Mean

$$n = \frac{\Sigma x}{n}$$

$$n = 5 \quad \Sigma x = 21 \quad \bar{x} = ?$$

$$\bar{x} = 4.2$$

Median

$$\tilde{x} = \left[\frac{n+1}{2} \right]^{th}$$

$$\tilde{x} = \left[\frac{5+1}{2} \right]^{th}$$

$$\tilde{x} = \left[\frac{6}{2} \right]^{th}$$

$$\tilde{x} = 3^{th}$$

$$\tilde{x} = 4$$

Mode

Mode = $\tilde{x}$ = most repeated value

= there is no repeated value so no mode

7. **A set of data contains the value as**

148, 145, 160, 157, 156, 160.

Show that Mode > Median > Mean.

Ans:

x	145	148	156	157	160	160	$\Sigma x = 926$
-----	-----	-----	-----	-----	-----	-----	------------------

mean

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\Sigma x = 926 \quad n = 6$$

$$\bar{x} = \frac{926}{6}$$

$$\bar{x} = 154.33$$

Median

$$= \tilde{x} = \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} \text{observation} + \left(\frac{n+2}{2} \right)^{\text{th}} \text{observation} \right]$$

$$= \tilde{x} = \frac{1}{2} \left[\left(\frac{6}{2} \right)^{\text{th}} + \left(\frac{6+2}{2} \right)^{\text{th}} \right]$$

$$= \tilde{x} = \frac{1}{2} [3^{\text{rd}} + 4^{\text{th}}] = \tilde{x} = \frac{1}{2} [156 + 157]$$

$$= \tilde{x} = \frac{1}{2}[313] = \tilde{x} = 156.5$$

Mode = the most repeated value

Mode = 160

Mode > Median > Mean

160 > 156.5 > 154.33

Hence Proved the condition

8. **The monthly attendance of 10 students for their lunch in the hostel is recorded as:**

21, 15, 16, 18, 14, 17, 15, 12, 13, 11.

Find the median and mode of the attendance. Also find the mean if $D = A - 20$.

Ans:

x	$D = x - 20$
11	$11 - 20 = -9$
12	$12 - 20 = -8$
13	$13 - 20 = -7$
14	$14 - 20 = -6$
15	$15 - 20 = -5$
15	$15 - 20 = -5$
16	$16 - 20 = -4$
17	$17 - 20 = -3$
18	$18 - 20 = -2$
21	$21 - 20 = 1$
	$\sum D = -48$

Mode = $\hat{x}$ = The most repeated value = 15

Median

$$= \tilde{x} = \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} \text{ observation} + \left(\frac{n+2}{2} \right)^{\text{th}} \text{ observation} \right]$$

$$= \tilde{x} = \frac{1}{2} \left[\left(\frac{10}{2} \right)^{\text{th}} + \left(\frac{10+2}{2} \right)^{\text{th}} \right]$$

$$= \tilde{x} = \frac{1}{2} [5^{\text{th}} + 6^{\text{th}}]$$

$$= \tilde{x} = \frac{1}{2} [15 + 15]$$

$$\tilde{x} = \frac{1}{2} (30)$$

$$\tilde{x} = 15$$

$$\text{Mean } \sum D = -48$$

$$\bar{x} = A + \frac{\sum D}{n} \quad n = 10$$

$$\bar{x} = 20 - \frac{48}{10}$$

$$\bar{x} = 20 - 4.8$$

$$\bar{x} = 15.2$$

9. **On a prize distribution day, 50 students brought pocket money as under:**

Rupees	5-10	10-15	15-20	20-25	25-30
Frequency (f)	12	9	18	7	4

1. **Find the median and mode of the above data.**

Ans:

Rupees	15-10	10-15	15-20	20-25	25-30	$\sum f = 50$
Frequency	12	$f_1 = 9$	$f_m = 18$	$f_2 = 7$	4	
C.F	12	21	39	46	50	

$$\text{Median} = \tilde{n} = \ell + \frac{h}{f} \left[\frac{n}{2} - c \right]$$

$$\frac{n}{2} = \frac{50}{2} = 25$$

$$\ell = 15 \quad f = 18 \quad h = 5 \quad c = 21$$

$$\tilde{n} = 15 + \frac{20}{18} \Rightarrow \tilde{n} = 15 + 1.11 \Rightarrow \tilde{n} = 16.11$$

$$\text{Mode} = \ell + \frac{(f_m - f_1)h}{(f_m - f_1) + (f_m - f_2)}$$

$$f_m = 18 \quad f_1 = 9 \quad f_2 = 7 \quad \ell = 15 \quad h = 5$$

$$\text{Mode} = x = 15 + \frac{(18-9)5}{(18-9) + (18-7)} = x = 15 + \frac{45}{20}$$

$$x = 15 + 2.25$$

$$x = 17.25$$

2. Find the arithmetic mean of the data given above using coding method.

Ans:

Rupees	Frequency	x	$U = \frac{x-A}{h}$	fu
5-10	12	7.5	$\frac{7.5-7.5}{5} = 0$	0
10-15	9	12.5	$\frac{12.5-7.5}{5} = \frac{5}{5} = 1$	9
15-20	18	17.5	$\frac{17.5-7.5}{5} = \frac{10}{5} = 2$	36
20-25	7	22.5	$\frac{22.5-7.5}{5} = \frac{15}{5} = 3$	21
25-30	4	27.5	$\frac{27.5-7.5}{5} = \frac{20}{5} = 4$	16
	$\sum f = 50$			= 76

Coding method

$$\sum f = 50 \quad \sum fu = 76 \quad h = 5 \quad A = 7.5$$

$$\bar{n} = A + \frac{\sum fu \times h}{\sum f} \Rightarrow \bar{n} = 7.5 + \frac{76 \times 5}{50}$$

$$\bar{n} = 7.5 + 7.6 \Rightarrow \bar{n} = 15.1$$

3. The arithmetic mean of the ages of 20 boys is 13 years, 4 months and 5 days. Find the sum of their ages. If one of the boys is of age exactly 15 years. What is the average age of the remaining boys?

Ans:

One of the boy age = 15
Sum of 19 boys = 9740 - 54785
= 91925

$$\text{Average age} = \frac{91925}{19} = 4835$$

= 4835 days
= 13 years 3 month 4 days

4. Calculate the arithmetic mean from the following information:

1. If $D = X - 140$, $\sum D = 500$ and $n = 10$

Ans:

$$\bar{x} = A + \frac{\sum D}{n}$$

$$\bar{x} = 140 + \frac{500}{10} \Rightarrow \bar{x} = 140 + 50$$

$$\bar{x} = 190$$

2. If $U = \frac{x-130}{6}$, $\sum U = -150$ and $n = 15$

Ans:

$$\bar{x} = A + \frac{\sum U \times h}{n}$$

$$A = 130 \quad \sum U = -150 \quad n = 15$$

$$\bar{x} = 130 - \frac{150 \times 6}{15} \Rightarrow \bar{x} = 130 - 60$$

$$\bar{x} = 70$$

3. If $D = x - 25$, $\sum fD = 300$ and $\sum f = 20$

Ans:

$$\bar{x} = A + \frac{\sum fD}{\sum f}$$

$$\bar{x} = 25 + \frac{300}{20}$$

$$\bar{x} = 40$$

4. If $U = \frac{x-120}{5}$, $\sum fU = 60$ and $\sum f = 100$

Ans:

$$A = 120 \quad h = 5$$

$$\bar{x} = 120 + \frac{60 \times 5}{100}$$

$$\bar{x} = 120 + \frac{300}{100} \Rightarrow \bar{x} = 120 + 3$$

$$\bar{x} = 123$$

5. The three children Haris, Maham and Minal made the following scores in a game conducted by a group of teachers in the school.

Haris scores	50	55	70	85	90
Maham scores	75	60	60	45	53
Minal scores	80	77	66	42	48

It is decided that the candidate who gets the highest average score will be

awarded rupees 1000. Who will get the awarded amount?

Ans:

Haris scores	Maham scores	Minal scores
50	75	80
55	60	77
70	60	66
85	45	42
90	53	48

$$\sum x = 350$$

$$\sum x = 293$$

$$\sum x = 313$$

$$\sum x = 350 \quad n = 5$$

$$\text{Harish } \bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{350}{5}$$

$$\bar{x} = 70$$

$$\text{Maham } \sum x = 293 \quad n = 5$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{293}{5}$$

$$\bar{x} = 58.6$$

$$\text{Minal } \sum x = 313 \quad n = 5$$

$$\text{Minal } \bar{x} = \frac{\sum x}{n} = \frac{313}{5} = 62.6$$

Haris will be awarded amount

6. Given below is a frequency distribution derived by making a substitution as $D = X - 20$. Calculate the arithmetic mean.

Ans:

D	f	fD
-6	1	-6
-4	3	-12
-2	6	-12
0	20	0
2	26	52
4	12	48
6	2	12
	$\sum f = 70$	$\sum fD = 82$

$$\bar{x} = A + \frac{\sum fD}{\sum f}$$

$$\bar{x} = 20 + \frac{82}{70}$$

$$\bar{x} = 20 + 1.17$$

$$\bar{x} = 21.17$$

7. Being partners Hafsa and Fatima took part in a quiz programme. They made the following number of points 45, 51, 58, 61, 74, 48, 46 and 50. Compute the average number of points using deviation $D = x - 58$

Ans:

x	$D = x - A$
45	$45 - 58 = -13$
51	$51 - 58 = -7$
58	$58 - 58 = 0$
61	$61 - 58 = 3$
74	$74 - 58 = 16$
48	$48 - 58 = -10$
46	$46 - 58 = -12$
50	$50 - 58 = -8$
	$\sum D = -31$

$$\bar{x} = A + \frac{\sum D}{n}$$

$$n = 8 \quad \sum D = -31 \quad A = 58$$

$$\bar{x} = A + \frac{\sum D}{n} \Rightarrow \bar{x} = 58 + \frac{(-31)}{8}$$

$$\bar{x} = 58 - 3.875 \Rightarrow \bar{x} = 54.125$$

8. A person purchased the following food items:

Food item	Quantity (in Kg)	Cost per Kg (in Rs.)
Rice	10	96
Flour	12	48
Ghee	4	190
Sugar	3	49
Mutton	2	650

What is the weighted mean of cost of food item per kg?

Ans:

Food item	Quantity (in Kg)	Cost per Kg (in Rs.)	Wx
Rice	10	96	960
Flour	12	48	576
Ghee	4	190	760
Sugar	3	49	147
Mutton	2	650	1300
	$\sum W = 31$		$\sum Wx = 3743$

$$\bar{x} = \frac{\sum Wx}{\sum W} \Rightarrow \bar{x} = \frac{3743}{31} = 120.74$$

9. For the following data, find the weighted mean.

Item	Quantity	Cost of item (in thousands)
Washing Machine	5	35
Heater	3	5
Stove	2	13
Dispenser	6	18

$$\bar{x} = \frac{\sum Wx}{\sum W}$$

$$\sum Wx = 5 \times 35 + 3 \times 5 + 2 \times 13 + 6 \times 18$$

$$\sum Wx = 175 + 15 + 26 + 108$$

$$\sum Wx = 324, \sum W = 16$$

$$\bar{x} = \frac{324}{16}$$

$$\bar{x} = 20.25$$

10. A company is planning its next year marketing budget across five years: yearly budgets (in million) are: 5, 7, 8, 6, 7. find the average budget for the next year.

Ans:

W	5	7	8	6	7	$\sum W = 33$
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$$n = 5 \quad \sum W = 33$$

$$\bar{x}_w = \frac{\sum W}{n} \Rightarrow \bar{x}_w = \frac{33}{5}$$

$$\bar{x}_w = 6.6$$

Average budget = 6.6 million

11. Ahmed obtained the following marks in a certain examination. Find the weighted mean if weights 5, 4, 2, 3, 2, 4 respectively are allotted to the subjects.

Urdu	English	Science	Math	Islamiyat	Computer
78	65	80	90	85	72

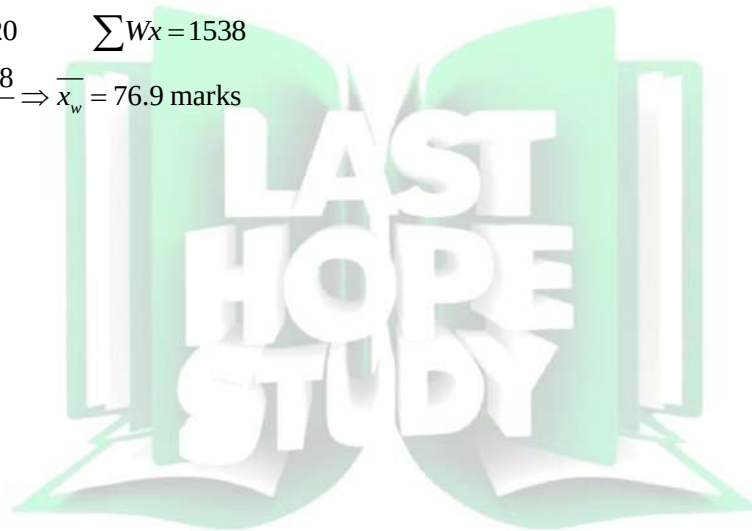
Ans:

	x	W	Wx
Urdu	78	5	390
English	65	4	260
Science	80	2	160
Math	90	3	270
Islamiyat	85	2	170
Computer	72	4	288
		$\sum W = 20$	$\sum Wx = 1538$

$$\bar{x}_w = \frac{\sum Wx}{\sum W}$$

$$\sum W = 20 \quad \sum Wx = 1538$$

$$\bar{x}_w = \frac{1538}{20} \Rightarrow \bar{x}_w = 76.9 \text{ marks}$$



REVIEW EXERCISE

- Four options are given against each statement. Encircle the correct one.

- Which data takes only some specific values?

(a) Continuous data (b) Discrete data
(c) Grouped data (d) Ungrouped data

- The number of times a value occurs in a data is called:

(a) Frequency (b) Relative frequency
(c) Class limit (d) Class boundaries.

- Midpoint is also known as:

(a) Mean (b) Median
(c) Class limit (d) Class Mark

- Frequency polygon is also drawn/constructed by using:

(a) Histogram (b) Bar graph
(c) Class boundaries (d) Class limit

- The difference between the greatest value and the smallest value is called:

(a) Class limits (b) Midpoint
(c) Relative frequency (d) Range

find out the _____ of a data set.

(a) Class boundaries (b) Cumulative frequency

(c) Middle or centre value (d) Frequency

- If the mean of 5, 7, 8, 9 and x is 7.5, what will be the value of x?

(a) 10 (b) 8
(c) 8.5 (d) 5.8

- Find the mode of the given data: 2, 5, 8, 9, 0, 1, 3, 7 and 10

(a) 5 (b) 7
(c) 0 (d) No mode

- In a data the values (observations) which appears or occurs most often is called:

(a) Mean (b) Mode
(c) Median (d) Weighted mean

- Find the median of the given data: 110, 125, 122, 130, 124, 127 and 120

(a) 124 (b) 120
(c) 125 (d) 127

Answer Key

1	b	2	a	3	d	4	a	5	d
6	c	7	c	8	d	9	b	10	a

- Measure of central tendency is used to
- Define the following

Ans:

- Frequency distribution**

A distribution or table that represents classes or groups along with their respective classes frequencies is called frequency distribution.

- Histogram (unequal class limits)**

This is a graph of adjacent rectangles constructed on xy plane. A histogram is similar to bar graph but it is constructed for a frequency distribution.

- Mean**

Is defined as a value of variable which is obtained by dividing the sum of all the values by their number of observations. The arithmetics means of a set of values $x_1, x_2, x_3, \dots, x_n$ is

denoted by $\bar{x}$ (read as x. bar) and is calculated as $\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\sum x}{n}$ direct method.

Where the sign Σ stands for the sum and n is the number of observations.

- Median**

Median is the middle most value in an arranged (ascending or descending order) data set median is denoted by $\tilde{x}$

Median ($\tilde{x}$) = $\left[\frac{n+1}{2} \right]^{th}$ observation (when n is odd number)

$$\text{Median } (\tilde{x}) = \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} \text{ observation} + \left(\frac{n+1}{2} \right)^{\text{th}} \text{ observation} \right] \text{ when even number}$$

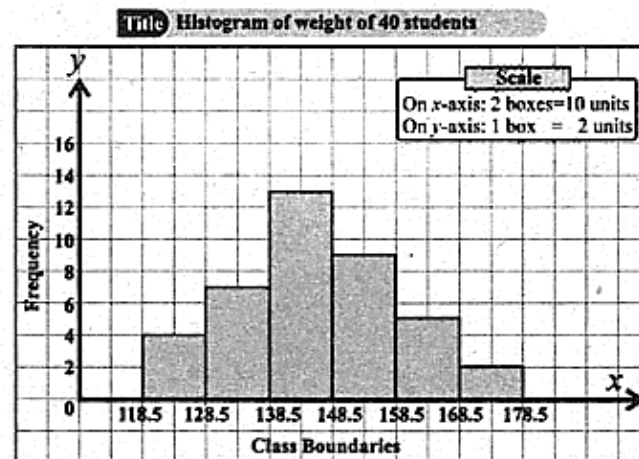
- Following are the weights of 40 students recorded to the nearest (lbs)
138,164,150,132,144,125,149,157,146,158,140,147,136,148,152,144,
168,126,138,176,163,119,154,165,146,173,174,147,135,153,140,135
161,145,135,142,150,156,145,128.
- Make a frequency table taking size of class limits as 10
 - Draw histogram.
 - Draw a frequency polygon of the given data.

Ans:

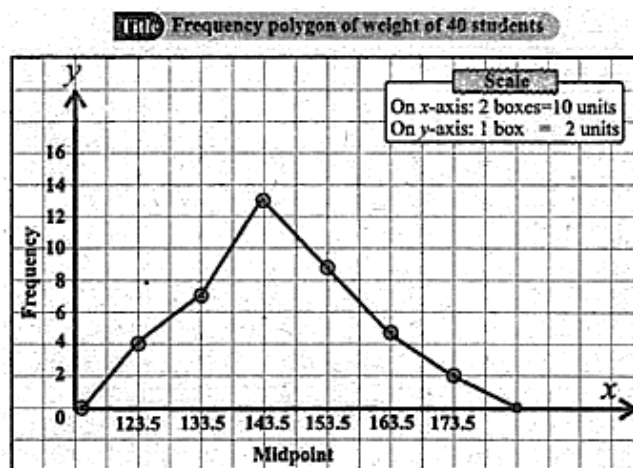
(a)

Class limit	Tally Marks	f	Mid point	Class Boundaries
119 – 128	IIII	4	123.5	118.5 – 128.5
129 – 138	IIII II	7	133.5	128.5 – 138.5
139 – 148	IIII III	13	143.5	138.5 – 148.5
149 – 158	IIII III	9	153.5	148.5 – 158.5
159 – 168	IIII	5	163.5	158.5 – 168.5
169 – 178	II	2	173.5	168.5 – 178.5

(b)



(c)

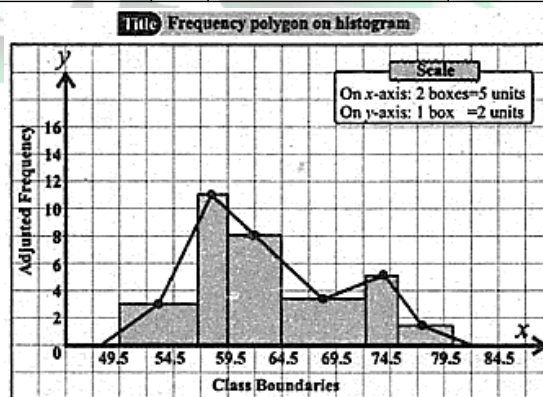


- From the table given below. Draw a frequency polygon on histogram for the given frequency distribution.

Weight (kg)	50–56	57–59	60–64	65–72	73–75	76–80
Frequency (f)	25	32	40	30	15	8

Ans:

Weight (kg)	f	Class boundaries	Mid point
50–56	25	49.5–56.5	53
57–59	32	56.5–59.5	58
60–64	40	59.5–64.5	62
65–72	30	64.5–72.5	68.5
73–75	15	72.5–75.5	73.5
76–80	8	76.5–80.5	78



- Given below are marks obtained by 45 students in the monthly test of Biology:

Marks	20–24	25–29	30–34	35–39	40–44	45–49
No. of students	05	08	12	15	03	02

With reference to the above table find the following:

Ans:

Marks	No. of students	Class boundaries
20 – 24	05	19.5 – 24.5
25 – 29	08	24.5 – 29.5
30 – 34	12	29.5 – 34.5
35 – 39	15	34.5 – 39.5
40 – 44	03	39.5 – 44.5
45 – 49	02	44.5 – 49.5

1. Upper class boundary of the 5th class.

Ans:

44

2. Lower class boundaries of all the classes.

Ans:

19.5, 24.5, 29.5, 34.5, 39.5, 44.5

3. Midpoint of all the classes.

Ans:

22, 27, 32, 37, 42, 47

4. The class interval with the least frequency.

Ans:

5

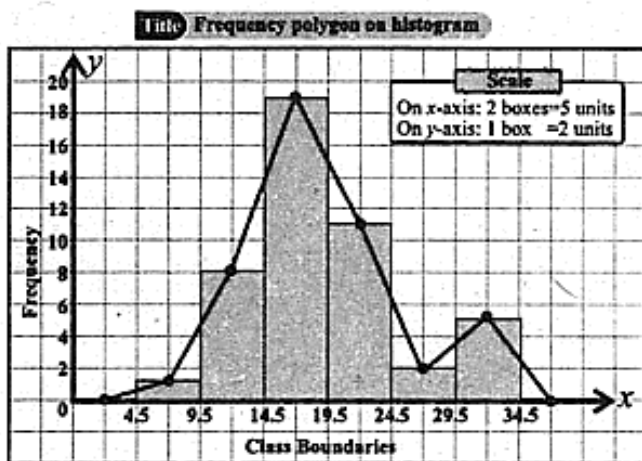
- Given below is frequency distribution.

Draw frequency polygon and histogram for the distribution.

Class limits	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34
Frequency	1	8	18	11	2	5

Ans:

Class limit	Frequency	Class boundarie	Midpoint
5 – 9	1	4.5 – 9.5	3.5
10 – 14	8	9.5 – 14.5	12
15 – 19	18	14.5 – 19.5	17
20 – 24	11	19.5 – 24.5	22
25 – 29	2	24.5 – 29.5	27
30 – 39	5	29.5 – 39.5	34.5



- For the following data, find the weighted mean.

Item	Quantity	Cost of item (Rs.)
Chair	20	500
Table	20	400
Black board	10	750
Tube light	25	230
Cupboard	09	950

Ans:

Item	Quantity (W)	Cost of item (Rs)	Wx
Chair	20	500	10000
Table	20	400	8000
Black board	10	750	7500
Tube light	25	230	5750
Cup board	09	950	8550
	$\sum W = 84$		$\sum Wx = 39800$

$$\bar{X}_w = \frac{\sum Wx}{\sum W}$$

$$\bar{X}_w = \frac{39800}{84}$$

$$\bar{X}_w = 47381$$

- A principal of a school allocates funds of Rs. 50, 500 to five different sectors:
- Chairs: Rs. 15000
 - Table: Rs. 12,000
 - Black boards: Rs. 6,000
 - Room renovation: Rs. 10,000
 - Gardening: Rs. 7,000

Find the average of funds allocation in each sector of the school.

Ans:

$$\sum x = 50,000$$

$$n = 5$$

$$\bar{x} = ?$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{50000}{5} = 10,000 \text{ Rs}$$

- The marks of a student Saad in six tests were 84, 91, 72, 68, 87, 78. Find the arithmetic mean of his marks:

Ans:

x	84	91	72	68	87	78	480
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$$\bar{x} = \frac{\sum x}{n}$$

$$n = 6 \quad \sum x = 480$$

$$\bar{x} = \frac{480}{6}$$

$$\bar{x} = 80 \text{ Marks}$$

- Adjoining distribution showed maximum load (in kg) supported by certain ropes. Find the mean load using short method.

Max - Load kg	93-97	98-102	103-107	108-112	113-117	118-122
No. of ropes	2	5	8	12	6	2

Ans:

Max load kg	No of ropes	x	D = x - A	fD
39-97	2	95	95-100 = -5	-10
98-102	5	100	100-100 = 0	0
103-107	8	105	105-100 = 5	40
108-112	12	110	110-100 = 10	120
113-117	6	115	115-100 = 15	90
118-122	2	120	1120-100 = 20	40
	35			$\sum fD = 280$

$$\sum f = 35 \quad \sum fD = 280$$

$$\bar{x} = A + \frac{\sum fD}{\sum f}$$

$$\bar{x} = 100 + \frac{280}{35}$$

$$x = 100 + 8$$

$$x = 108 \text{ kg}$$

- Usman rolled a fair dice eight times. Each time their sum was recorded as 8, 5, 6, 6, 9, 4, 3, 11. Find the median and mode of the sum.

Ans:

x	3	4	5	6	6	8	9	11
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Mode = The most repeated value = 6

$$\text{Median} = \frac{1}{2} \left[\left(\frac{n}{2} \right) \text{term} + \left(\frac{n+2}{2} \right) \text{term} \right]$$

$$= \frac{1}{2} \left[\left(\frac{8}{2} \right) \text{term} + \left(\frac{8+2}{2} \right) \text{term} \right] = \frac{1}{2} [4^{\text{th}} + 5^{\text{th}}]$$

$$= \frac{1}{2} [6 + 6] = \frac{1}{2} [12] = 6$$

- Two partners Mr. Aslam and Mrs. Kalsoom run a company. In the following data the weekly wages (in Rs.) of employees who work in the company are given:

Wages (Rs.)	600 – 700	700 – 800	800 – 900	900 – 1000	1000 – 1100
Employees	3	5	7	21	11

Ans:

Wages (Rs)	Employees (f)	x	fx	CF
600 – 700	3	650	1950	3
700 – 800	5	750	3750	8
800 – 900	7 f ₁	850	5950	15
900 – 1000	21 f _m	950	19950	36
1000 – 1100	11 f ₂	1050	11550	47
	47		$\sum fx = 43150$	

$$\text{Mean } \bar{x} = \frac{\sum fx}{\sum f}$$

$$\bar{x} = \frac{43150}{47}$$

$$\bar{x} = 918.085$$

$$\bar{x} = 918.09$$

$$\bar{x} = \ell + \frac{h}{f} \left[\frac{n}{2} - c \right]$$

$$\frac{n}{2} = \frac{47}{2} = 23.5$$

Median

$$\ell = 900 \quad f = 21 \quad c = 15 \quad h = 100$$

$$\tilde{x} = \ell + \frac{h}{f} \left[\frac{n}{2} - c \right]$$

$$\tilde{x} = 900 + \frac{100}{21} [23.5 - 15]$$

$$\tilde{x} = 900 + \frac{100}{21} [8.5]$$

$$\tilde{x} = 900 + 40.47$$

$$\tilde{x} = 940.47$$

$$\text{Mode} = \hat{x} = \ell + \frac{(f_m - f_1)h}{(f_m - f_1) + (f_m - f_2)}$$

$$\ell = 900 \quad f_m = 21 \quad f_1 = 7 \quad f_2 = 11 \quad h = 100$$

$$\hat{x} = 900 + \frac{(21 - 7)}{(21 - 7) + (21 - 11)}$$

$$\hat{x} = 900 + \frac{14 \times 100}{14 + 10} = 900 + \frac{1400}{24}$$

$$= 900 + 58.33 = 958.33$$

