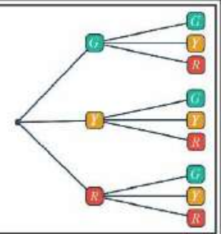
13 UNIT PROBABILITY



In our daily life, we normally say that manufacturing companies give warranty on their products, there is chance that some product might not meet warranty time period. A person judges the chances of winning cricket match of a team based on previous performances etc. All the above statements have lack prediction with certainty. In such situations, what makes it easier for us to represent the chance of an event occurring numerically i.e., probability.

History!
The word "probability" is derived from the Latin word "Probabilitas". It means "probity". Girolamo Cardano is known as the father of probability. He was an Italian doctor and mathematician.

Hence, Probability is the chance of occurrence ofa particular event. Probability is calculated by using the given formula:

$$\text{Probability}\,\frac{\text{Number of favourable ourtcomes}}{\text{Total number of possible outcomes}}$$

It is written as: $P(A)=\dfrac{n(A)}{n(S)}$

$P(A)$ =Probability of an event A

$n(A)$ =Number of favorable outcomes

$n(S)$ =Total number of possible outcomes

Basic Concepts of Probability:

Experiment:

The process which generates results e.g., tossing a coin, rolling a dice, etc. is called an experiment.

Outcomes:

The results of an experiment are called outcomes e.g., the possible outcomes of tossing a coin are head or tail, the possible outcomes of rolling a dice are 1, 2, 3, 4, 5, or 6.

Favorable Outcome:

An outcome which represents how many times we expect the things to be happened e.g., while tossing a coin, there is 1 favorable outcome of getting head or tail. While rolling a dice, there are 3 favorable outcomes of getting multiples of 2 i.e. {2, 4, 6}

Sample space:

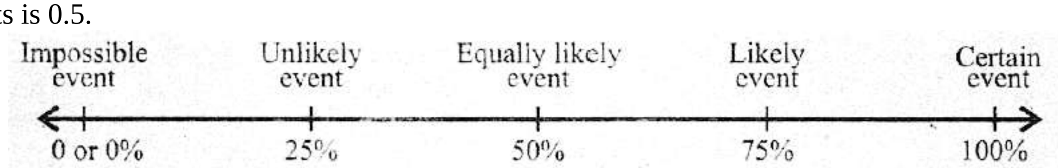
The set of all possible outcomes of an experiment is called sample space. It is denoted by 'S' e.g., while tossing a coin, the sample space will be S = {H, T}.
While rolling a dice, the sample space will be S = {1, 2, 3, 4, 5, 6}.

Event:

The set of results of an experiment is called an event e.g., while rolling a dice getting even number is an event i.e., A = {2, 4, 6}; $n(A)$ = 3.

Recall! Types of Events:

1. **Certain event:** An event which is sure to occur. The probability of sure event is 1.
2. **Impossible event:** An event cannot occur in any trial. The probability of this event is 0,
3. **Likely event:** An event which will probably occur. It has greater chance to occur.
4. **Unlikely event:** An event which will not probably occur. It has less chance to occur.
5. **Equally likely events:** The events which have equal chance of occurrence. The probability of these events is 0.5.



Probability of Single Event

Example 1:

Abdul Raheem rolls a fair dice, what is the probability of getting the number divisible by 3?

Solution: When a dice is rolled, the sample space will be:

S = {1, 2, 3, 4, 5, 6} ; $n(S) = 6$

Let "A" be the event of getting the number divisible by 3.

$A = \{3,6\}; n(A) = 2$

$P(A)\frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

The probability of getting the number divisible by 3 is $\frac{1}{3}$

Keep in mind
The range of probability for an event is
$0 \le P(A) \le 1$

Teachers note:
Clear the concept of all the types of events by using different colours of balls or pencils etc.

Example 2:

If Zeeshan rolled two fair dice, find the probability of getting:

Even numbers on both dice.

Multiples of 3 on both dice.

Even number on the first dice and the number 3 on the second dice.

At least the number 3 on the first dice and number 4 on the second dice.

Solution:

When a pair of fair dice is rolled, the sample space will be:

1st / 2nd	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Try Yourself
Can you find out the sample space when 3 dice are rolled.

(i) Even numbers on both dice.

Let *"A"* be the event of getting even numbers on both dice.

A = {(2,2), (2,4), (2,6), (4,4), (4,6), (6,2), (6,4), (6,6)

$n(A) = 9; n(S) = 36$

$A(A) = \frac{n(A)}{n(S)} = \frac{9}{36} = \frac{1}{4}$

Thus, the probability of getting even numbers on both dice is $\frac{1}{4}$.

(ii) Multiple of 3 on both dice.

Let "B" be the event of getting multiples of 3 on both dice.

$B = \{(3,3),(3,6),(6,3),(6,6)\}$

$n(B) = 4; n(S) = 36$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

Thus, the probability of getting multiples of 3 on both dice is $\frac{1}{9}$.

Probability of an Event Not Occurring

Sometimes, we are interested in the probability that the head will not occur while tossing a coin.

Let “*A*” be the event of getting head while tossing a coin, then the event “A” be the event of not getting head while tossing a coin.

The probability of not getting head while tossing a coin is known as the complement of that event. It is written as or P (A’) or P (A^c).

The complement of an event “A” is calculated by the given formula:

P(A’) = 1 – P(A)

For example, while tossing a coin, the probability of getting a head is

$P(A)\frac{1}{2}$.

Teachers note
Give more examples to explain complement of events e.g., if the desired outcome is head on a flipping coin, the complement is tail. The compliment rule states that the sum of the probability -of an event and its complement must be equal to l.

And the probability of not getting a head is

$$P(A')=1-P(A) = 1-\frac{1}{2}=\frac{1}{2}$$

Thus, the complement of the event of getting a head is $\frac{1}{2}$.

Example 3:

Zubair rolls a dice, what is the probability of not getting the number 6?

Solution:

Let “*A*” be the event of getting the number 6.

The sample space while rolling a dice is : $S=\{1,2,3,4,5,6\}$

$n(S)=6$

$A=\{6\};n(A)=1$

$P(A)=\frac{n(A)}{n(S)}=\frac{1}{6}$

To find out probability of not getting the number 6, we have

$P(A')=1-P(A)=1-\frac{1}{6}=\frac{6-1}{6}=\frac{5}{6}$

Thus, the probability of not getting the number 6 is $\frac{5}{6}$.

Example 4:

If two fair dice are rolled. What is the probability of getting:

(i) not a double six (ii) not the sum of both dice is 8

Solution:

Sample space of two fair dice is given by

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

$P(A)=\frac{n(A)}{n(S)}$

(i) not a double six

Let “A” be the event that a double six occurs.

$A=\{(6,6)\};n(A)=1$

$P(A)=\frac{n(A)}{n(S)}=\frac{1}{36}$

Let " A' " be the event that not a double six occurs

As we know that

$P(A) = 1 - P(A) \; = 1 - \frac{1}{36} = \frac{36-1}{36} = \frac{35}{36}$

Thus, the probability of not getting the double six is $\frac{35}{36}$.

(ii) not the sum of both dice is 8.

Let "B" be the event that the sum of both dice is 8.

$n(B) = 5$

Let "B'" be the event not sum of both dice is 8.

$P(B') = 1 - P(B) \; = 1 - \frac{5}{36} = \frac{36-5}{36} = \frac{31}{36}$

Thus, the probability of not the sum of both dice be 8 is $\frac{31}{36}$.

Real Life Problems Involving Probability:

Example 5:

Let A; B and C are three missiles and they are fired at a target. If the probabilities of hitting the target are $P(A)\frac{1}{4}, P(B) = \frac{3}{7}, P(C) = \frac{5}{9}$, respectively.

Find the probabilities of

(i) missile A does not hit the target (ii) missile B does not hit the target.

(iii) missile C does not hit the target

Solution:

(i) Missile A does not hit the target.

Since, $P(A)\frac{1}{4}$

Let 'A' be the event that missile A does not hit the target

$P(A') = 1 - P(A) \; = 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$

Thus, the probability of missile 'A' does not hit the target is $\frac{3}{4}$.

(ii) Missile 'B' does not hit the target.

Since, P(B) = $\frac{3}{7}$

Let 'B' be the event missile B does not hit the target

$P(B') = 1 - P(B) \qquad = 1 - \frac{3}{7} \; = \frac{7-3}{7} = \frac{4}{7}$

Thus, the probability of missile 'B' does not hit the target is $\frac{4}{7}$.

(iii) Missile 'C' does not hit the target.

Since, $P(C') = 1 - P(C) \; = 1 - \frac{5}{9} = \frac{9-5}{9} = \frac{4}{9}$

Thus, the probability of missile 'C' does not hit the target is $\frac{4}{7}$.

EXERCISE 13.1

1. Arshad rolls a dice, with sides labelled L, M, N, O, P, U. What is the probability that the dice lands on consonant?

Ans:

Number of letters $= n(S) = 6$

Number of consonants

$= n(A) = 6 - 2 = 4$ L, M, N, P

Formula $P(A)=\frac{n(A)}{n(S)}=\frac{4}{6}=\frac{2}{3}$

2. Shazia throws a pair of fair dice. What will be the probability of getting:

Ans:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

1. Sum of dots is at least 4.

$(1,3),(1,4),(1,5),(1,6),(2,2),(2,3),(2,4),(2,5),(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3)$
$(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,6)$

No of favorable out comes $=n(A)=33$

$P(A)=\frac{n(A)}{n(S)}=\frac{33}{36}=\frac{11}{12}$

2. Product of the both dots is between 5 to 10.

$(1,5),(1,6),(2,3),(2,4),(2,5),(5,1),(3,2),$
$(3,3)(4,2),(5,2),(6,1)$

$n(S)=36$

$n(A)=11$

$P(A)=\frac{n(A)}{n(S)}=\frac{11}{36}$

3. **The difference between both the dots is equal to 4.** 0.0042

Ans:

$(1,5),(2,6),(5,1),(6,2)$

are favorable outcomes are

$n(S)=36$

$n(A)=4$

$P(A)=\frac{n(A)}{n(S)}=\frac{4}{36}=\frac{1}{9}$

4. Number at least 5 on the first dice and the number at least 4 on the second dice.

Ans:

Favorable outcome 5 and 4

outcome of 5 = 2 $(5,6)$

$n(A)=2$

$n(S)=5$

$P(A)=\frac{n(A)}{n(S)}$

$P(A)=\frac{2}{6}$

$P(A)=\frac{1}{3}$

Favorable out of $4=3(4,5,6)$

$n(S)=6$

$n(A)=3$

$P(A)=\frac{N(A)}{N(S)}=\frac{3}{6}=\frac{1}{2}$

Probability = probability of getting 5 × Probability of 4

$=\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)=\frac{1}{6}$

5. One alphabet is selected at random from the word “MATHEMATICS”. Find the probability of getting:

1. Vowel

Ans:

Total letters in MATHEMATICS

$=n(S)=11$

$n(A)=4$

$P(A)=\frac{n(A)}{n(S)}=\frac{4}{11}$ $=11-4=7$

2. Consonant

Ans:

$n(A)=7$

$n(S)=11$

$P(A)=\frac{n(A)}{n(S)}=\frac{7}{11}$

3. A E

Ans:

$$n(A)=\frac{1}{11}$$
$$n(S)=11$$
$$P(A)=\frac{1}{11}$$

4. An A

Ans:

$$n(A)=2$$
$$n(S)=11$$
$$P(A)=\frac{n(A)}{n(S)}$$
$$P(A)=\frac{1}{11}$$

5. Not M

Ans:

$$n(S)=11$$
$$n(A)=11-2=9$$
$$P(A)=\frac{n(A)}{n(S)}$$
$$P(A)=\frac{9}{11}$$

6. Not T

Ans:

$$n(A)=11-2=9$$
$$n(S)=11$$
$$P(A)=\frac{n(A)}{n(S)}=\frac{9}{11}$$

6. Aslam rolled a dice. What is the probability of getting the numbers 3 or 4? Also find the probability of not getting the number 3 or 4.

Ans:

Possible outcome $n(S)=6$

Favorable outcome of 3 or 4 $=n(A)=2$

$$P(A)=\frac{n(A)}{n(S)}$$
$$P(A)=\frac{2}{6}=\frac{1}{3}$$

Probability of getting the number 3 or 4

$$P(A')=1-P(A)=1-\frac{1}{3}=\frac{3-1}{3}=\frac{2}{3}$$

7. Abdul Hadi labelled cards from 1 to 30 and put them in a box. He selects a card at random. What is the probability that selected card containing:

1. The number 25

Ans:

$$n(S)=30$$
$$n(A)=1$$
$$P(A)=\frac{n(A)}{n(S)}=\frac{1}{30}$$

2. Number between 17 to 22

Ans:

$17,18,19,20,21,22$

$$n(S)=30$$
$$n(A)=6$$
$$P(A)=\frac{n(A)}{n(S)}$$
$$P(A)=\frac{6}{3}=\frac{1}{5}$$

3. Number at least 20

Ans:

$20,21,22,23,24,25,26,27,28,29,30$

$$n(A)=11$$
$$n(S)=30$$
$$P(A)=\frac{n(A)}{n(S)}=\frac{11}{30}$$

4. Number not 27and 29

Ans:

Total outcomes $=n(S)=30$

Number not 27 and $29=30-2=28$

$$n(A)=28$$
$$P(A)=\frac{n(A)}{n(S)}$$
$$P(A)=\frac{28}{30}$$
$$P(A)=\frac{14}{15}$$

5. Number not between 12 to 15

Ans:

Favorable out comes $=30-4=26$

$n(A)=26$

$n(S)=30$

$P(A)=\frac{n(A)}{n(S)}$

$P(A)=\frac{26}{30}$

$P(A)=\frac{13}{15}$

6. The probability that Ayesha will pass the examination is 0.85. What will be the probability that Ayesha will not pass the examination?

Ans:

Probability of passing = 0.85
Probability of not passing = ?
Probability of not passing $=1-P(A)$
Probability of not passing = 1–0.85 = 0.15

7. Taabish tossed a fair coin and rolled a fair dice once. Find the probability of the following events:

7. Tail on coin and at least on dice.

Ans:

Probability of getting a tail on the coin $=\frac{1}{2}$

Probability of getting at least 4 on the dice
There are 6 possible outcomes $1,2,3,4,5$ and 6
The favorable outcomes are 4, 5 and 6
So, the probability of getting at least 4 on the dice $=\frac{3}{6}$

$P(B)=\frac{1}{2}$

Probability of event $=P(B)$

Probability of getting a tail on coin at least 4 on the dice $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

8. Head on coin and the number 2, 3 on dice.

Ans:

Probability of getting a tail on the coin $=\frac{1}{2}$

There are 6 possibilities out comes $1,3,3,4,5$ and 6
The favorable outcomes are 2 and 3
Probability of getting 2 or 3 on the dice $=\frac{2}{6}=\frac{1}{3}$

Multiply both probabilities
Probabilities of getting a head on the coin and 2 or 3 on dice $=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$

9. Head and tail on coin and the number 6 on dice.

Ans:

Favorable out comes =2

Probability $=\frac{2}{2}=1$

Number 6 on dice $=1$

Probability $=\frac{1}{6}$

Required $=1\times\frac{1}{6}=\frac{1}{6}$

10. Not tail on coin and the number 5 on dice.

Ans:

Probability of getting a head in the coin $=\frac{1}{2}$

There are 6 possible outcomes $1,2,3,4,5$ and 6
The favorable outcome is 5

The probability of getting a 5 on dice $=\frac{1}{6}$

Multiply both probability together
Probability of not getting a tail on the coin and getting a 5 on the dice $=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$

11. Not head on coin and the number 5 and 2 on dice.

Ans:

Probability of getting a tail on the coin $=\frac{1}{2}$

There are 6 possible outcomes, $1,2,3,4,5$

and 6

Favorable outcomes are 2 and 5

The probability of getting a 5 or 2 on the dice $=\frac{2}{6}=\frac{1}{3}$

Multiply both probability

Probability of not getting head on the coin and getting as 5 or on the dice $=\frac{3}{2}+\frac{2}{3}=\frac{1}{2}+\frac{1}{3} =\frac{3+2}{6}=\frac{5}{6}$

8. A card is selected at random from a well shuffled pack of 52 playing cards. What will be the probability of selecting?

Ans:

1. A queen

Ans:

$P(A)=\text{Probability } =\frac{\text{Number of queen}}{\text{Number of cards}}$

$=\frac{n(A)}{n(S)}$

$=n(A)=4$

$=n(S)=52$

$=\frac{4}{52} =\frac{1}{3}$

2. Neither a queen nor a jack

Ans:

Probability of selecting a queen or a jack

$=\frac{\text{Number of queen and jack}}{\text{Total number of cards}} =\frac{8}{52}=\frac{2}{13}$

Probability of selecting neither a queen nor a jack = 1– Probability of selecting queen

$=1-\frac{2}{13} =\frac{13-2}{13} =\frac{11}{13}$

3. A card is chosen at random from a pack of 52 playing cards. Find the probability of getting:

Ans:

1. A jack

Ans:

A jack

Number of jack = 4

$\text{Probability}=\frac{\text{Number of jack}}{\text{Number of cards}}$

So probability of getting of jack

$=\frac{4}{52}=\frac{1}{13} =\frac{4}{52} =\frac{1}{3}$

2. No diamond

Ans:

Given a standard deck of 52 cards. We have to find the probability of getting no diamond there are 13 cards of diamond in a 52 card deck

$\text{Probability}=\frac{\text{Number of diamond card}}{\text{Total number of cards}}$

Probability of getting no diamond

$=1-\frac{13}{52} =\frac{52-13}{51}=\frac{39}{52}=\frac{3}{4}$

Relative Frequency as an Estimate of Probability:

Relative frequency tells us how often a specific event occurs relative to the total number of frequency event or trials. It is calculated by using the following method:

$$\text{Relative frequency} = \frac{\text{Frequency of specific event}}{\text{Total frequency}} = \frac{x}{N}, \text{Where } N = \sum f$$

Example8:

Find the relative frequency of the given date.

X	2	3	4	5	6	7	8
f	3	5	6	9	10	8	2

Solution:

X	f	**relative frequency**
2	3	$\frac{3}{43} = 0.07$
3	5	$\frac{5}{43} = 0.12$
4	6	$\frac{6}{43} = 0.14$
5	9	$\frac{9}{43} = 0.21$
6	10	$\frac{10}{43} = 0.23$
7	8	$\frac{8}{43} = 0.19$
8	2	$\frac{2}{43} = 0.04$
Total	$\sum f = 43$	

Real Life Application of Relative Frequency

Example 9:

A survey was conducted on 80 students of Grad-IX and asked about their favourite colour. The responses are:

(i) Red colour = 23 students

(ii) Green colour = 15 students

(iii) Pink colour = 25 students

(iv) Blue colour = 10 students

(v) White colour = 7 students.

> **Keep in mind**
> The sum of all the relative frequencies is always equal to approximately equal to 1.

Find the relative frequency for each colour.

Solution:

Total number of students $=80$

3. Relative frequency for red colour $=\dfrac{23}{80}=0.29$

It means that 29% students prefer red colour.

4. Relative frequency for green colour $=\dfrac{15}{80}=0.19$

It means that 19% students prefer green colour.

5. Relative frequency for pink colour $=\dfrac{25}{80}=0.31$

It means that 31% students prefer pink colour.

6. Relative frequency for blue colour $=\dfrac{10}{80}=0.12$

It means that 12% students prefer blue colour.

7. Relative frequency for white colour $=\dfrac{7}{80}=0.09$

It means 9% students prefer white colour.

Remember!
Relative frequency is an estimated probability of an event occurring when an experiment is repeated a fixed number of times.

Try Yourself!
Out of 200 students in a school, 80 play cricket, 50 play football, 25 play volleyball and 45 do not play any game. Can you find out the probability of the students who do not play any game and relative frequency of the students who play

EXERCISE 13.2

1. **A researcher collected data on a number of deaths from Horse-Ricks in Russian Army crops over to years. The table is as follows.**

No. of death	0	1	2	3	4	5	6
Frequency	60	50	87	40	32	15	10

Find the relative frequency of the given data.

Ans:

No. of death	frequency	rf
0	60	$\dfrac{60}{294}=\dfrac{30}{147}$
1	50	$\dfrac{50}{294}=\dfrac{25}{294}$
2	87	$\dfrac{87}{294}=\dfrac{29}{94}$
3	40	$\dfrac{40}{294}=\dfrac{20}{294}$
4	32	$\dfrac{32}{294}=\dfrac{16}{147}$
5	15	$\dfrac{15}{294}=\dfrac{5}{294}$
6	10	$\dfrac{10}{294}=\dfrac{5}{294}$
	294	

2. **The frequency of defective products in 750 samples are shown in the following table. Find**

the relative frequency for the given table.

No. od defectives per sample	0	1	2	3	4	5	6	7	8
No. of sample	120	140	94	85	105	50	40	66	50

Ans:

No. of defective per sample	No. of sample	*rf*
0	120	$\frac{120}{750}=\frac{4}{25}$
1	140	$\frac{140}{750}=\frac{47}{375}$
2	94	$\frac{94}{750}=\frac{47}{375}$
3	85	$\frac{85}{750}=\frac{17}{150}$
4	105	$\frac{105}{750}=\frac{21}{150}$
5	50	$\frac{50}{750}=\frac{1}{15}$
6	40	$\frac{40}{750}=\frac{4}{75}$
7	66	$\frac{66}{750}=\frac{33}{150}$
8	50	$\frac{50}{750}=\frac{1}{15}$
	750	

3. A quiz competition on general knowledge is conducted. The number of corrected answers out of 5 questions for 100 sets of questions is given below.

X	0	1	2	3	4	5
f	10	23	15	25	18	9

Find the relative frequencies for the given data.

Ans:

x	f	rf
0	10	$\frac{10}{100}=\frac{1}{10}$
1	23	$\frac{23}{100}=\frac{23}{100}$
2	15	$\frac{15}{100}=\frac{3}{20}$
3	25	$\frac{25}{100}=\frac{1}{4}$
4	18	$\frac{18}{100}=\frac{9}{50}$
5	9	$\frac{9}{100}=\frac{9}{100}$
	100	

4. **A survey was conducted from the 50 students of a class and asked about their favorite food. The responses are as under:**

Name of food item	Biryani	Fresh juice	Chicken	Bar. B.Q	Sweets
No. of students	40	07	21	15	25

Ans:

No. of food item	No. of students
Biryani	40
Fresh juice	7
Chicken	21
Bar B Q	15
Sweets	25
	108

- **How many percentages of students like biryani?**

Ans:

Biryani $=\frac{40}{108}\times 100=37\%$

- **How many percentages of students like chicken?**

Ans:

Chicken $=\frac{21}{108}\times 100=19.4=20\%$

- **Which food is the least like by the students?**

Ans:

Least like food by the children is fresh juice

- **Which food is the most prefer by the students?**

Ans:

Most like food by the children in biryani

5. **In 500 trials of a thrown of two dice, what is expected frequency that the sum will be greater than 8?**

Ans:

When two dice are thrown there are 36 possible outcomes
Here are the outcomes who the sum is greater than 8.

Sum 9: $(3,6)(4,5)(5,4)(6,3)$ = 4 outcomes

Sum 10: $(4,6)(5,5)(6,4)$ = outcomes 3

Sum 11: $(5,6)(6,5)$ = 2 outcomes

Sum 12 $=(6,6)$

Total outcomes with sum $>8=4+3+2+1=10$

Probability of sum $>8=\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$

$=\frac{10}{36}=\frac{5}{8}$

Expected frequency in 500 trials = Probability × number of trail

$=\frac{5}{18}\times 500=138.88=139$

6. What is the expectation of a person who is to get Rs. 120 if he obtains at least 2 heads in single toss of three coins?

Ans:

1. HHH (3 heads)
2. HHT (2 heads 1 tail)
3. HTH (2 heads 1 tail)
4. THH (2 heads 1 tail)
5. HTT (1 heads 2 tail)
6. THT (1 heads 2 tail)
7. TTH (1 heads 2 tail)
8. TTT (Three tail)

At least two heads

1. HHH (2 head, 1 tail)
2. HHT (2 head, 1 tail)
3. HTH (2 head, 1 tail)
4. THH (2 head, 1 tail)

There are 4 favorable outcomes

$$\text{Probability} = \frac{\text{favorable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{4}{8} = \frac{1}{2}$$

Expectation is the product of the probability and the amount $= \frac{1}{2} \times 120 = 60\,\text{Rs}$

5. Find the expected frequencies of the given data if the experiment is repeated 200 times.

x	0	1	2	3	4	5	6
$P(x)$	0.11	0.21	0.17	0.18	0.09	0.17	0.07

Ans:

x	$p(x)$	f
0	0.11	$0.11 \times 200 = 22$
1	0.21	$0.21 \times 200 = 42$
2	0.17	$0.17 \times 20 = 34$
3	0.18	$0.18 \times 200 = 36$
4	0.09	$0.09 \times 200 = 18$
5	0.17	$0.17 \times 200 = 34$
6	0.07	$0.07 \times 200 = 14$
		200

6. The probability of getting 5 sixes while tossing six dice is $\frac{2}{5}$, the dice is rolled 200 times. How many times would you expect it to show 5 sixes?

Ans:

Given probability $= \frac{2}{5}$

Expected frequency in 200 trails = probability × number of trials

$$= \frac{2}{5} \times 200 = 80 \text{ times}$$

REVIEW EXERCISE

- **Four options are given against each statement. Encircle the correct option.**
  - **Each element of the sample space is called:**
    (a) Event (b) Experiment
    (c) Sample point (d) Outcomes
  - **An outcome which represents how many times we expect the things to be happened is called:**
    (a) Outcomes
    (b) Favourable outcome
    (c) Sample space
    (d) Sample point
  - **Which one tells us how often a specific event occurs relative to the total number of frequency event or trails?**
    (a) Expected frequency
    (b) Sum of relative frequency
    (c) Relative frequency
    (d) Frequency
  - **Estimated probability of an event occurring is also known as:**
    (a) Relative frequency
    (b) Expected frequency
    (c) class boundaries
    (d) Sum of expected frequency
  - **The sum, of all expected frequencies is equal to the fixed number of:**
    (a) Trails
    (b) Relative frequencies
    (c) Outcomes
    (d) Events
  - **The chance of occurrence of a particular event is called:**
    (a) Sample space
    (b) Estimated probability
    (c) Probability
    (d) Expected frequency
  - **An event which will probably occur. It has greater chance to occur is called:**
    (a) Equally likely event (b) Likely event
    (c) Unlikely event
    (d) Certain event
  - **Find out the total number of possible sample space when 4 dice are rolled:**
    (a) 6^2 (b) 6^3
    (c) 6^4 (d) 6^6
  - **While rolling a pair of dice, what will be the probability of double 2?**
    (a) $\frac{1}{6}$ (b) $\frac{1}{3}$
    (c) $\frac{5}{6}$ (d) $\frac{1}{36}$
  - **A card is chosen from a pack of 52 playing cards, find the probability of getting no jack and king:**
    (a) $\frac{2}{13}$ (b) $\frac{11}{13}$
    (c) $\frac{2}{52}$ (d) $\frac{11}{52}$

Answer Key

1	c	2	b	3	c	4	a	5	a
6	c	7	b	8	c	9	d	10	b

- **Define the following**

1. Relative frequency

Ans:

Relative frequency is an estimated probability occurring when an experiment is repeated a fixed number of times

2. Expected frequency

Ans:

Expected frequency is a measure that estimate how often an even should be occurred depended on a probability expected frequency is found by using the following method.

Expected frequency = Total number of trials × Probability of event

$= N \times P(A)$

- **An urn contains 10 red balls, 5 green balls and 8 blue balls. Find the probability of selecting at random.**

1. A green ball

Ans:

$n(A) = 5$

$n(S) = 23$

$P(A) = \frac{n(A)}{n(S)}$

$P(A) = \frac{5}{23}$

2. A red ball

Ans:

$$n(A)=10$$
$$n(S)=23$$
$$n(A)=\frac{n(A)}{n(S)}$$
$$n(A)=\frac{10}{23}$$

3. A blue ball

Ans:

$$n(A)=8$$
$$n(S)=23$$
$$n(A)=\frac{n(A)}{n(S)}$$
$$n(A)=\frac{8}{23}$$

4. Not a red ball

Ans:

$$n(A)=5+8=13$$
$$n(S)=23$$
$$n(A)=\frac{n(A)}{n(S)}$$
$$n(A)=\frac{13}{23}$$

5. Not a green ball

Ans:

$$n(A)=10+8=18$$
$$n(S)=23$$
$$n(A)=\frac{n(A)}{n(S)}$$
$$n(A)=\frac{18}{23}$$

- **Three coins are tossed together. What is the probability of getting:**

1. Exactly thee heads

1. *HHH*
2. *HHT*
3. *HTH*
4. *THH*
5. *HTT*
6. *THT*
7. *TTH*
8. *TTT*

Only one outcomes *HHH*

$$\text{Probability}=\frac{\text{Number of favorable}}{\text{Total number of outcomes}}=\frac{1}{8}$$

9. At least two tails

Ans:

1. *HTT*
2. *THT*
3. *TTH*
4. *TTT*

$$\text{Probability}=\frac{\text{Number of favorable}}{\text{Total number of outcomes}}$$
$$=\frac{4}{8}=\frac{1}{2}$$

5. Not at least two heads

Ans:

(iii) *HHH*
(iv) *HHT*
(v) *HTH*
(vi) *THH*
(vii) *HTT*
(viii) *THT*
(ix) *TTH*
(x) *TTT*

Outcomes with at least two heads

(xi) *HHT*
(xii) *HHH*
(xiii) *HTH*
(xiv) *THH*

Outcomes with not at least two heads

(xv) *HTT*
(xvi) *THT*
(xvii) *TTH*
(xviii) *TTT*

$$\text{Probability}=\frac{\text{Number of favorable}}{\text{Total number of outcomes}}$$
$$=\frac{4}{8}=\frac{1}{2}$$

6. Not at least two heads

Ans:

Outcomes with not exactly two head

1. *HTT*
2. *THT*
3. *TTH*
4. *TTT*

Outcomes with exactly two heads

1. *HHT*
2. *HTH*
3. *THH*

$$\text{Probability}=\frac{\text{Number of favorable}}{\text{Total number of outcomes}}=\frac{5}{8}$$

- **A card is drawn from a wall shuffled pack of 52 playing cards. What will be the probability of getting:**

1. King or jack of red colour

Ans:

So, there are 2 kings of red color and 2 (jacks) of red color
Total favorable outcomes = 2 (kings) + 2 (jacks) =4
Total possible outcomes = 52

$$\text{Probability} = \frac{\text{Number of favorable}}{\text{Total number of outcomes}}$$

$$= \frac{4}{52} = \frac{1}{13}$$

2. Not “2: of club and spade

Ans:

There is 1 suit of clubs and 1 card 2 of clubs
There is 1 suit of spades and 1 card 2 of spades
Total cards that are 2 of club or 2 of spades = 2
Total cards that are not 2 of club or 2 or spades = 52 – 2 = 50

$$\text{Probability} = \frac{\text{Number of favorable}}{\text{Total number of outcomes}}$$

$$= \frac{50}{52} = \frac{25}{26}$$

- **Six coins are tossed 600 times. The number of occurrence of tails are recorded and shown in the table given below:**

No. of tails	0	1	2	3	4	5	6
Frequency	110	90	105	80	76	123	16

Find the relative frequency of given table.

Ans:

No. of tails	frequency	Relative freqeuency
0	110	$\frac{110}{600} = \frac{11}{60}$
1	90	$\frac{90}{600} = \frac{3}{20}$
2	105	$\frac{105}{600} = \frac{7}{40}$
3	80	$\frac{80}{600} = \frac{2}{15}$
4	75	$\frac{75}{600} = \frac{19}{150}$
5	123	$\frac{128}{600} = \frac{41}{200}$
6	16	$\frac{16}{600} = \frac{2}{75}$
	600	

- **From a lot containing 25 items, 8 items are defective. Find the relative frequency of non-defective items, also find the expected frequency of non defective items**

Ans:

Number of defective items = 8
Number of non defective items

$$= \frac{\text{Number of non defective item}}{\text{Number of item}}$$

$$= \frac{17}{25} = 0.68$$

So, find expected frequency of non defective items we need to know the probability of an item being non defective.

Probability of an item being non defective

$$= \frac{\text{Number of non defective item}}{\text{Total number of item}} = \frac{17}{25}$$

Expected frequency of non defective items

= Probability of an item being non defective × Total number of items

$$= \frac{17}{25} \times 25 = 17$$